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Practice article

lBest-HS algorithm based concurrent L_1 adaptive control for non-Linear systems

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HIGHLIGHTS

- Concurrent mode of hybridization of meta-heuristic optimization technique with the L_1 adaptive control structure for process automation.
- Choice of L_1 adaption gains according to the merit of the adaptable parameters.
- The guaranteed stability of the proposed design with the superior convergence performance of lbest HS algorithm based L_1 adaptive controller.

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ABSTRACT

This paper proposes a new approach for designing stable hybrid L_1 adaptive controller employing lbest topological model of harmony search (HS) algorithm. The proposed design approach guarantees desired stability and simultaneously provides satisfactory tracking performance for a class of non-linear systems. The design methodology for the controller utilizes the meta-heuristic global search feature of HS algorithm and the local search phenomenon of L_1 adaptive control strategy in tandem. The paper also analytically describes the superiority of lbest topological model compared to the conventional HS algorithm in terms of convergence phenomenon, when hybridized with L_1 adaptive control. The proposed hybrid control methodology has been implemented for benchmark simulation case studies and real-time experimentation to demonstrate its usefulness.

1. Introduction

Designing control strategies for non-linear systems are drawing much attention to the researchers over last few decades. There are several theoretical and practical ways like feedback linearization, gain scheduling which had been utilized and successfully employed to tackle nonlinearity in many applications [1]. But, in such cases, at the time of linearization of the system, some information will fly away and high variation of uncertainties cannot be mitigated by these types of controllers. Backstepping, gain scheduling, self-tuning regulators etc. are also extensively employed for designing the adaptive controllers [2,3]. In recent years, a number of promising adaptive control schemes are proposed and experimentally verified with different types of non-linear systems. A decentralized adaptive control scheme is proposed for a class of interconnected nonlinear systems [2] without requiring any *a priori* knowledge for the control directions of its subsystems. Here, Nussbaum gain is employed which produces high overshoot, chattering problem and poor dynamic performance. To mitigate the chattering problem during the operation of the nonlinear systems, adaptive back-stepping control strategy with hysteretic quantizer is utilized in [3]. To overcome the drawbacks of the direct adaptive control, as utilized in [2] and [3], a conjuncture of the input to state stable (ISS) feedback and the model-free extremum seeking (ES) algorithm based indirect adaptive control scheme was proposed in [4] for the nonlinear systems with time-varying parametric uncertainties. For the nonlinear systems with unknown actuator delay, prediction based delay adaptive control scheme has been proposed in [5]. These controllers provide unsatisfactory transient responses to reduce instability of the system. Other than the classical approaches, intelligent control schemes such as fuzzy logic [6,7], neural network [8,9] etc. are also utilized for different classes of non-linear systems. Though increase in time complexity due to the training process of the controller and/ or network is the main disadvantage of these cases. Model reference adaptive control (MRAC) is another popular adaptive control scheme for non-linear systems. However MRAC is very much dependent on system initial conditions and the reference signals.

To avoid those problems, L_1 adaptive controller [10,11] is introduced to take care of uncertainty in system parameters and external disturbances without sacrificing the stable transient performance. Here, perfect cancellation of uncertainties and disturbances are achieved by keeping the control signal within

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the control channel bandwidth and it is independent of initial condition and references. The L_1 adaptive control strategy utilizes a state predictor of the original non-linear system, high gain adaptation rules and a filter to achieve satisfactory transient performance as well as high robustness. It thus guarantees the robust transient performance along with fast adaptation to the system with uncertainties and disturbances. In recent past, L_1 adaptive control strategy finds applications in many fields like aircraft control [12,13], maximum power point tracking in small wind energy conversion system [14], pico-scale satellite test bed system [15], healthcare [16] etc. Despite of being advantageous, the conventional L_1 adaptive control strategy has some distinctive disadvantages. The designer should employ a judicious trade-off between the robust performance and fast adaptation during the design process [10,11]. It also requires *a priori* knowledge of controller parameters and the design does not guarantee the optimal/near optimal solution [10,11]. Although, the adaptation gains for different parameters of design process are same, their influences on the control law are different. It sometime produces unnecessary high values of design variables and in turn, high filter gain to obtain a physically realizable control law. Moreover, the free parameters of L_1 adaptive control scheme are selected manually in a trivial manner. This may cause substantial erroneous controller design when the external disturbances are large enough.

In this work, L_1 adaptive control architecture and a meta-heuristic optimization, namely harmony search (HS) algorithm, are hybridized in a concurrent manner to utilize the advantages of L_1 adaptive control scheme, and at the same time avoiding its drawbacks. The novelty of this proposed scheme is that, it does not require any *a priori* knowledge about the system to be controlled. Here, the manual tuning of design parameters are replaced by the optimal/near optimal solution by the utilization of HS algorithm. The adaptation gains are chosen as different values for different adaptable parameters depending on their relative importance. The operating principle of the proposed work is that, the L_1 adaptive control framework is first tuned in an offline manner to get the adaptation laws for unknown constant, uncertainties and disturbances present in the system utilizing the predictor output. Then, the tuned controller is employed online to control the system. Thus, the predictor will not come into play during the online operation, when the performance of the tuned controller is evaluated. So in a nutshell, this paper proposes a tuning methodology for the L_1 adaptive control framework with optimization technique.

The essential characteristic of a good optimization technique is its balance in exploration and exploitation of candidate solutions during the search process. The basic HS algorithm, proposed by Z. W. Geem [17], suffers from the exploration point of view although it shows very good exploitation phenomenon in different applications [18,19]. An improved group improvisation based HS algorithm, basically a local best (lbest) model of HS algorithm, proposed by Das Sharma [20], poses a good balance between exploration and exploitation due to its intense harmony improvisation technique. In this paper, the exploration and exploitation capability of lbest HS algorithm has been mathematically analysed, to show its better convergence than its conventional counterpart. Moreover, the stability analysis of the control scheme along with the meta-heuristic optimization technique has been performed explicitly. The Lyapunov stability theory for an adaptive controller and the spectral radius convergence of a meta-heuristic optimization technique are put together to analyse the stability of the proposed lbest HS based optimal L_1 adaptive controller.

Thus, the main contributions of the proposed methodology pointed out as:

- Concurrent mode of hybridization of meta-heuristic optimization technique with the L_1 adaptive control structure for process automation.
- Choice of L_1 adaption gains according to the merit of the adaptable parameters.
- The guaranteed stability of the proposed design with the superior convergence performance of lbest HS algorithm based L_1 adaptive controller.

The proposed hybrid L_1 adaptive control scheme is compared with lbest HS based control scheme without L_1 adaptive controller, classical L_1 adaptive control scheme [10,11,21], PID augmented L_1 adaptive controller [22], fuzzy feedback filter L_1 adaptive controller [23], basic HS algorithm based hybrid design and particle swarm optimization (PSO) based hybrid design. The results obtained from the simulation case studies and an extensive hardware case study show that the proposed lbest HS algorithm based hybrid L_1 adaptive control scheme outperforms the other competing design techniques, as mentioned above.

This paper is organized as follows. Description of L_1 adaptive controller with its stability proof is given in Section 2. Section 3 details the stochastically optimized hybrid L_1 adaptive controller design. Simulation and experimental case studies are given in Section 4. Section 5 concludes the paper with further research directions.

2. Problem formulation and L_1 controller design

Conventional adaptive controllers are mainly used for the systems with small uncertainties and in general, are having unsatisfactory transient performance. Controller setting is very much dependent upon the system initial states, reference inputs and sometime it leads to instability of the system [10]. For highly non-linear system and with uncertainties, large enough, conventional adaptive controller fails to stabilize the system. In addition, these controllers produce high frequency control signal, large transient errors and also cannot deal with time varying systems. A new variant of adaptive control strategy, namely L_1 adaptive controller is computationally fast and robust in adaptation [10,11]. It can also handle large time varying uncertainties even during transient period.

As shown in Fig. 1, a typical L_1 adaptive controller comprises of mainly three parts viz. state predictor, controller and adaptation law. The predictor produces similar output as system and it predicts the uncertainties present in the system. Adaptation law adapts that uncertainties and control law produces control signal for both system and its predictor. System input is given through a low pass filter where the higher order frequencies are filtered out.

2.1. Problem formulation

Let us consider the dynamics of a single input single output (SISO) system as:

$$\dot{\underline{x}}(t) = A_m\underline{x}(t) + b(\omega u(t) + \underline{\theta}^T(t)\underline{x}(t) + \sigma(t)), \underline{x}(0) = \underline{x}_0 \quad (1)$$

$$y(t) = c^T\underline{x}(t) \quad (2)$$

where, $\underline{x}(t) \in \Re^n$ is the state vector, $u(t) \in \Re$ is the input to the system, $y(t) \in \Re$ is the controlled output, $b \in \Re^{n\times 1}$ and $c \in \Re^{1\times n}$ are known constants vector, $A_m \in \Re^{n\times n}$ is system matrix, Hurwitz in nature.$\omega \in \Re$ is unknown constant related to system, $\underline{\theta}(t) \in \Re^n$ is parameter uncertainty vector and $\sigma(t) \in \Re$ is time varying disturbance of the system model. These ω, $\underline{\theta}(t)$ and $\sigma(t)$ are bounded by compact convex sets $\Omega \in \Re$, $\underline{\Theta} \in \Re^n$, and $\Sigma \in \Re$ respectively $\forall t \geq 0$. Now, the objective is to design a robust

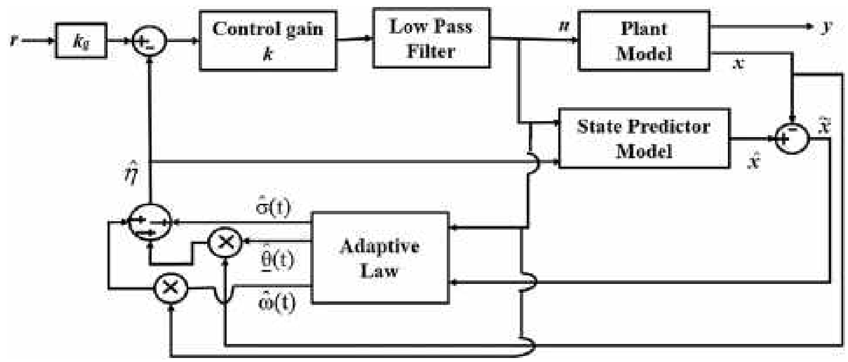

Fig. 1. Architecture of L_1 adaptive controller.

controller that can give a bounded control signal such that the system output can properly track reference signal $r(t)$, both in transient and steady state period keeping all other error signals bounded.

Predictor design: Now, in the L_1 adaptive control configuration a predictor is employed to estimate the system states with the help of the adaptive estimates of the modelled uncertainties and disturbances. The state predictor model is considered as [21,24]:

$$\dot{\hat{\underline{x}}}(t) = A_m\hat{\underline{x}}(t) + b(\hat{\omega}(t)u(t) + \hat{\underline{\theta}}^T(t)\underline{x}(t) + \hat{\sigma}(t)) \tag{3}$$

where, $\hat{\underline{x}}(t) \in \Re^n$ is the state estimation. $\hat{\omega}(t) \in \Re$, $\hat{\underline{\theta}}(t) \in \Re^n$, and $\hat{\sigma}(t) \in \Re$ are the adaptive estimates of unknown parameter, uncertainties and disturbances present in the system respectively. The error between the system state and predictor state i.e. $\tilde{\underline{x}}(t) = \hat{\underline{x}}(t) - \underline{x}(t)$ is utilized to formulate the adaptation laws, given as follows:

$$\dot{\hat{\omega}}(t) = \Gamma_1 \mathrm{Proj}(\hat{\omega}(t), -\tilde{\underline{x}}^T(t)Pb\, u(t)) \tag{4}$$

$$\dot{\hat{\underline{\theta}}}(t) = \Gamma_2 \mathrm{Proj}(\hat{\underline{\theta}}(t), -\underline{x}(t)\tilde{\underline{x}}^T(t)Pb) \tag{5}$$

$$\dot{\hat{\sigma}}(t) = \Gamma_3 \mathrm{Proj}(\hat{\sigma}(t), -\tilde{\underline{x}}^T(t)Pb) \tag{6}$$

where, P is the solution of the algebraic Lyapunov equation of the system given by $A_m{}^T P + PA_m = -Q$, where Q is an arbitrary symmetric matrix satisfying $Q = Q^T > 0$. $\underline{\Gamma} = [\Gamma_1\Gamma_2\Gamma_3] > 0$ is the adaption gain vector with different adaptive gain components. Proj $(\cdot,\cdot)$ denotes the Pomet–Praly projection operator [25].

The adaptation laws given in (4)–(6) are obtained from the condition to minimize the state error between system state and predictor state i.e. $\tilde{\underline{x}}(t) = \hat{\underline{x}}(t) - \underline{x}(t)$ by solving the Lyapunov stability criterion.

Let, the Lyapunov candidate function is as follows:

$$v = \frac{1}{2}\tilde{\underline{x}}^T(t)P\tilde{\underline{x}}(t) + \frac{1}{2}\tilde{\omega}^T(t)\Gamma_1{}^{-1}\tilde{\omega}(t) + \frac{1}{2}\tilde{\underline{\theta}}^T(t)\Gamma_2{}^{-1}\tilde{\underline{\theta}}(t) + \frac{1}{2}\tilde{\sigma}^T(t)\Gamma_3{}^{-1}\tilde{\sigma}(t) \tag{7}$$

where, $\tilde{\omega}(t) = \hat{\omega}(t) - \omega$ is the error due to system modelling, $\tilde{\underline{\theta}}(t) = \hat{\underline{\theta}}(t) - \underline{\theta}(t)$ is the error vector due to parameter uncertainty and $\tilde{\sigma}(t) = \hat{\sigma}(t) - \sigma(t)$ is the error due to modelling of disturbances in the system.

The dynamics of the state error between system state and predictor state is given as follows:

$$\dot{\tilde{\underline{x}}}(t) = A_m\tilde{\underline{x}}(t) + b\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right] \tag{8}$$

Taking the time derivative of (7) it can be written:

$$\begin{aligned}\dot{v} = {} & \frac{1}{2}\dot{\tilde{\underline{x}}}^T(t)P\tilde{\underline{x}}(t) + \frac{1}{2}\tilde{\underline{x}}^T(t)P\dot{\tilde{\underline{x}}}(t) + \frac{1}{2}\dot{\tilde{\omega}}^T(t)\Gamma_1{}^{-1}\tilde{\omega}(t) \\ & + \frac{1}{2}\tilde{\omega}^T(t)\Gamma_1{}^{-1}\dot{\tilde{\omega}}(t) + \frac{1}{2}\dot{\tilde{\underline{\theta}}}^T(t)\Gamma_2{}^{-1}\tilde{\underline{\theta}}(t) \\ & + \tfrac{1}{2}\tilde{\underline{\theta}}^T(t)\Gamma_2{}^{-1}\dot{\tilde{\underline{\theta}}}(t) + \tfrac{1}{2}\dot{\tilde{\sigma}}^T(t)\Gamma_3{}^{-1}\tilde{\sigma}(t) + \frac{1}{2}\tilde{\sigma}^T(t)\Gamma_3{}^{-1}\dot{\tilde{\sigma}}(t)\end{aligned} \tag{9}$$

Now, as ω, $\underline{\theta}(t)$, and $\sigma(t)$ are bounded within a compact set of prescribed minimum and maximum values as $[\omega_{\min}, \omega_{\max}] \in \Omega$ $[\underline{\theta}_{\min}, \underline{\theta}_{\max}] \in \Theta$ and $[\sigma_{\min}, \sigma_{\max}] \in \Sigma[\sigma_{\min}, \sigma_{\max}] \in \Sigma$, where, $\omega_{\min}, \underline{\theta}_{\min}, \sigma_{\min}$ are the minimum and $\omega_{\max}, \underline{\theta}_{\max}, \sigma_{\max}$ are the maximum values of ω, $\underline{\theta}(t)$ and $\sigma(t)$ respectively. Therefore, the following upper bounds hold:

$\hat{\omega}(t) - \omega \le \hat{\omega}(t) - \omega_{\min}$, or, $\dot{\tilde{\omega}}(t) \le \dot{\hat{\omega}}(t)$. Similarly, $\dot{\tilde{\underline{\theta}}}(t) \le \dot{\hat{\underline{\theta}}}(t)$ and $\dot{\tilde{\sigma}}(t) \le \dot{\hat{\sigma}}(t)$. Putting these bounds and value from (8) in (9), the upper bound of $\dot{v}$ becomes [21],

$$\begin{aligned}\dot{v} \le {} & \frac{1}{2}\tilde{\underline{x}}^T(t)A_m{}^T P\tilde{\underline{x}}(t) + \frac{1}{2}\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right]^T b^T P\tilde{\underline{x}}(t) \\ & + \frac{1}{2}\tilde{\underline{x}}^T(t)PA_m\tilde{\underline{x}}(t) + \frac{1}{2}\tilde{\underline{x}}^T(t)Pb\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right] \\ & + \tilde{\omega}^T(t)\Gamma_1{}^{-1}\dot{\hat{\omega}}(t) + \tilde{\underline{\theta}}^T(t)\Gamma_2{}^{-1}\dot{\hat{\underline{\theta}}}(t) + \tilde{\sigma}^T(t)\Gamma_3{}^{-1}\dot{\hat{\sigma}}(t)\end{aligned} \tag{10}$$

or,

$$\begin{aligned}\dot{v} \le {} & \frac{1}{2}\left[\tilde{\underline{x}}^T(t)A_m{}^T P\tilde{\underline{x}}(t) + \tilde{\underline{x}}^T(t)PA_m\tilde{\underline{x}}(t)\right] \\ & + \frac{1}{2}\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right]^T b^T P\tilde{\underline{x}}(t) \\ & + \frac{1}{2}\tilde{\underline{x}}^T(t)Pb\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right] + \tilde{\omega}^T(t)\Gamma_1{}^{-1}\dot{\hat{\omega}}(t) \\ & + \tilde{\underline{\theta}}^T(t)\Gamma_2{}^{-1}\dot{\hat{\underline{\theta}}}(t) + \tilde{\sigma}^T(t)\Gamma_3{}^{-1}\dot{\hat{\sigma}}(t)\end{aligned}$$

or,

$$\begin{aligned}\dot{v} \le {} & \frac{1}{2}\tilde{\underline{x}}^T(t)\left[A_m{}^T P + PA_m\right]\tilde{\underline{x}}(t) \\ & + \frac{1}{2}\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right]^T b^T P\tilde{\underline{x}}(t) \\ & + \frac{1}{2}\tilde{\underline{x}}^T(t)Pb\left[\tilde{\omega}(t)u(t) + \tilde{\underline{\theta}}^T(t)\underline{x}(t) + \tilde{\sigma}(t)\right] \\ & + \tilde{\omega}^T(t)\Gamma_1{}^{-1}\dot{\hat{\omega}}(t) + \tilde{\underline{\theta}}^T(t)\Gamma_2{}^{-1}\dot{\hat{\underline{\theta}}}(t) + \tilde{\sigma}^T(t)\Gamma_3{}^{-1}\dot{\hat{\sigma}}(t)\end{aligned} \tag{11}$$

As, $A_m{}^T P + PA_m = -Q$ and $\underline{\widetilde{x}}^T(t)Pb = b^T P\underline{\widetilde{x}}(t)$, thus,

$$\dot{v} \le -\frac{1}{2}\underline{\widetilde{x}}^T(t)Q\underline{\widetilde{x}}(t) + \left[\underline{\widetilde{x}}^T(t)Pb\widetilde{\omega}(t)u(t) + \widetilde{\omega}^T(t)\Gamma_1{}^{-1}\dot{\hat{\omega}}(t)\right] + \left[\underline{\widetilde{x}}^T(t)Pb\underline{\widetilde{\theta}}^T(t)\underline{x}(t) + \underline{\widetilde{\theta}}^T(t)\Gamma_2{}^{-1}\dot{\underline{\hat{\theta}}}(t)\right] + \left[\underline{\widetilde{x}}^T(t)Pb\widetilde{\sigma} + \widetilde{\sigma}^T(t)\Gamma_3{}^{-1}\dot{\hat{\sigma}}(t)\right] \quad (12)$$

Now, to ensure the stability of the closed-loop system in the sense of Lyapunov, it should have,

$$\dot{v} \le -\frac{1}{2}\underline{\widetilde{x}}^T(t)Q\underline{\widetilde{x}}(t) \le 0 \quad (13)$$

Thus, to achieve the inequality in (13) the following conditions should hold:

$$\dot{\hat{\omega}}(t) = -\Gamma_1\underline{\widetilde{x}}^T(t)Pb\,u(t)$$

$$\dot{\underline{\hat{\theta}}}(t) = -\Gamma_2\underline{\widetilde{x}}^T(t)Pb\underline{x}(t)$$

$$\dot{\hat{\sigma}}(t) = -\Gamma_3\underline{\widetilde{x}}^T(t)Pb$$

Hence, the closed-loop stability of the system is guaranteed, even with the presence of uncertainties and disturbances.

Now, to ensure the boundedness of the adaptable variables, projection operator is used and the adaptation laws (4)–(6) are obtained, where, the Pomet–Praly projection operator is defined as [25]:

$$\mathrm{Proj}(\alpha, y) = \begin{cases} y - \dfrac{\nabla f(\alpha)(\nabla f(\alpha))^T}{\|\nabla f(\alpha)\|^2} y f(\alpha), \\ \quad \text{if} f(\alpha) > 0 \text{ and } y^T\nabla f(\alpha) > 0 \\ y, \quad \text{otherwise} \end{cases} \quad (14)$$

Remark 1. The rates of adaptation of unknown constant, time varying uncertainties and time varying disturbances are chosen differently depending on their requirement to stabilize the system.

Remark 2. The projection bounds for unknown constant, time varying uncertainties and time varying disturbances are so chosen that the control signal reside within the actuator saturation limit. The design process chooses the L_1 adaptive control parameters in such a way that, the designed controller provides optimal/ near optimal performance with bounded control signal.

Remark 2 further infers that a constrained optimization scheme is required to implement the design criteria during the experimental study.

Control law: The L_1 adaptive control law is generated as follows [21,22]:

$$u(s) = -kC(s)\left[\hat{\eta}(s) - k_g r(s)\right] \quad (15)$$

where, $\hat{\eta}(s)$ is the Laplace transform of

$$\hat{\eta}(t) = \underline{\hat{\theta}}^T(t)\underline{x}(t) + \hat{\sigma}(t) + \hat{\omega}(t)u(t) \quad (16)$$

k is a positive feedback gain, k_g is the feed-forward pre-filter gain to $r(t)$ and $r(s)$ is Laplace transform of $r(t)$. $D(s)$ must be chosen such that $C(s) = \frac{\rho D(s)}{1+\rho D(s)}$ is a strictly proper and stable transfer function, with low-pass filter gain $C(0) = 1$ and cut-off frequency ρ. A simple choice for getting a low pass filter is $D(s) = \frac{1}{s}$. Low pass filter ensures the rejection of uncertainties within the bandwidth of the control channel as well as high frequencies introduced due to the high adaptation rate. Inclusion of low pass filter in control signal decouples the adaptation and robustness properties of the system [24,26–30]. The advantage of this L_1 adaptive controller in comparison with MRAC is the inclusion of this filtering technique that permits guaranteed satisfactory transient performance for systems output.

Now, Let, $H(s)$ is a strictly proper stable transfer function such as $H(s) = (s\mathrm{I} - \mathrm{A}_m)^{-1}\, b$.

For L_1 adaptive controller a state predictor of a strictly proper stable system containing unknown constant, time varying disturbances and time varying uncertainties can be viewed as a low pass system. Taking the Laplace transform of predictor equation (3) becomes,

$$\hat{x}(s) = H(s)(C(s) - 1)\hat{\eta}(s) + H(s)C(s)k_g r(s)$$

or,

$$\hat{x}(s) = \overline{G}(s)\hat{\eta}(s) + G(s)r(s), \quad (17)$$

$$\overline{G}(s) = H(s)(C(s) - 1) \quad (18)$$

$$G(s) = k_g H(s)C(s) \quad (19)$$

Eqs. (18) and (19) represent the predictor and plant model respectively.

Here, the system will have the filtered output and rest of the high frequency signal will be delivered to the predictor model [10].

State boundedness: Let, $\lambda_{\min}(P)$ be the minimum Eigenvalue of P. Then the following upper bound

$$\lambda_{\min}(P)\left\|\underline{\widetilde{x}}(t)\right\|^2 \le \underline{\widetilde{x}}^T(t)P\underline{\widetilde{x}}(t) \le v(t) \le v(0), \forall t \ge 0 \quad (20)$$

holds which implies that,

$$\left\|\underline{\widetilde{x}}(t)\right\|^2 \le {}^{v(0)}/_{\lambda_{\min}(P)}, \forall t \ge 0 \quad (21)$$

where, $v(t)$ is appropriate Lyapunov function.

L_∞ norm defines that,

$$\left\|\underline{\widetilde{x}}(t)\right\|_{L\infty} = \max_{i=1,2,\ldots,n,t\ge 0} |\widetilde{x}_i(t)| \quad (22)$$

Using (22) in (21) ensures that,

$$\max_{i=1,2,\ldots,n,t\ge 0} |\widetilde{x}_i(t)| \le \sqrt{{}^{v(0)}/_{\lambda_{\min}(P)}}, \forall t \ge 0 \quad (23)$$

Therefore, $\forall t > 0$,

$$\left\|\underline{\widetilde{x}}(t)\right\|_{L\infty} \le \sqrt{{}^{v(0)}/_{\lambda_{\min}(P)}} \quad (24)$$

From triangular relationship for norms gives:

$$\left|\left\|\underline{\hat{x}}(t)\right\|_{L\infty} - \left\|\underline{x}(t)\right\|_{L\infty}\right| \le \sqrt{{}^{v(0)}/_{\lambda_{\min}(P)}} \quad (25)$$

Again from projection algorithm it follows that,

$$\hat{\omega}(t) \in \Omega,\ \underline{\hat{\theta}}(t) \in \Theta,\ \hat{\sigma}(t) \in \Sigma; \forall t \ge 0 \quad (26)$$

As all the adaptation parameters are bounded, therefore, from the definition of $\hat{\eta}(t)$ in (16), we have,

$$\left\|\hat{\eta}(t)\right\|_{L\infty} \le \omega_{\max}\left\|u(t)\right\|_{L\infty} + \underline{\theta}_{\max}\left\|\underline{x}(t)\right\|_{L\infty} + \sigma_{\max} \quad (27)$$

where, $\omega_{\max} = \max_{\omega\in\Omega}\omega$, $\underline{\theta}_{\max} = \max_{\underline{\theta}\in\underline{\Theta}}\sum_{i=1}^{n}|\theta_i|$, θ_i is the ith element of $\underline{\theta}$ and $\sigma_{\max} = \max_{\sigma\in\Sigma}\sigma$. Substituting $\|x_i\|_{L\infty}$ value from Eq. (25) in Eq. (27) gives,

$$\left\|\hat{\eta}(t)\right\|_{L\infty} \le \omega_{\max}\left\|u(t)\right\|_{L\infty} + \underline{\theta}_{\max}\left(\left\|\underline{\hat{x}}(t)\right\|_{L\infty} + \sqrt{{}^{v(0)}/_{\lambda_{\min}(P)}}\right) + \sigma_{\max} \quad (28)$$

Now, from Eq. (17) and L_∞ norm condition,

$$\left\|\underline{\hat{x}}(t)\right\|_{L\infty} \le \left\|\overline{G}(s)\right\|_{L1}\left\|\hat{\eta}(t)\right\|_{L\infty} + \left\|G(s)\right\|_{L1}\left\|r(t)\right\|_{L\infty} \quad (29)$$

Putting the value of $\left\|\hat{\eta}(t)\right\|_{L\infty}$ from (28) in (29) yields that,

$$\begin{aligned}\left\|\hat{\underline{x}}(t)\right\|_{L\infty} &\leq \left\|\overline{G}(s)\right\|_{L1}\omega_{\max}\left\|u(t)\right\|_{L\infty}\\ &+ \left\|\overline{G}(s)\right\|_{L1}\underline{\theta}_{\max}\left(\left\|\hat{\underline{x}}(t)\right\|_{L\infty}+\sqrt{v(0)/\lambda_{\min}(P)}\right)\\ &+ \left\|\overline{G}(s)\right\|_{L1}\sigma_{\max}+\left\|G(s)\right\|_{L1}\left\|r(t)\right\|_{L\infty}\end{aligned} \tag{30}$$

Let,

$$\lambda = \left\|\overline{G}(s)\right\|_{L1}\underline{\theta}_{\max} \tag{31}$$

From L_1 gain requirement k, kg, $C(s)$ are designed to satisfy $\left\|\overline{G}(s)\right\|_{L1}\underline{\theta}_{\max} < 1$.

Therefore, the relationship in (30) can be modified as:

$$(1-\lambda)\left\|\hat{\underline{x}}(t)\right\|_{L\infty} \leq \left\|\overline{G}(s)\right\|_{L1}\omega_{\max}\left\|u(t)\right\|_{L\infty}+\lambda\sqrt{v(0)/\lambda_{\min}(P)} + \left\|\overline{G}(s)\right\|_{L1}\sigma_{\max}+\left\|G(s)\right\|_{L1}\left\|r(t)\right\|_{L\infty}$$

or,

$$\left\|\hat{\underline{x}}(t)\right\|_{L\infty} \leq \frac{\left\|\overline{G}(s)\right\|_{L1}\omega_{\max}\left\|u(t)\right\|_{L\infty}+\lambda\sqrt{\frac{v(0)}{\lambda_{\min}(P)}} + \left\|\overline{G}(s)\right\|_{L1}\sigma_{\max}+\left\|G(s)\right\|_{L1}\left\|r(t)\right\|_{L\infty}}{(1-\lambda)} \tag{32}$$

Since, $v(0)$, $\lambda_{\min}(P)$, $\left\|\overline{G}(s)\right\|_{L1}$, $\left\|G(s)\right\|_{L1}$, $\left\|r(t)\right\|_{L\infty}$, λ, $\left\|u(t)\right\|_{L\infty}$, $\omega_{\max}$, $\sigma_{\max}$ all are finite, and $\lambda < 1$, this equation implies that $\left\|\hat{\underline{x}}(t)\right\|_{L\infty}$ is finite for any $t > 0$, hence $\hat{\underline{x}}(t)$ is bounded.

The relationship in (25) states that $\left\|\underline{x}(t)\right\|_{L\infty}$ is also finite $\forall t > 0$, and therefore $\underline{x}(t)$ is bounded.

Therefore, if the unknown constant, time varying uncertainties and disturbances related to the system are present simultaneously, then, the satisfactory transient performance as well as overall stability of the L_1 adaptive controller are guaranteed precisely.

Now using (13), as Q is positive definite, therefore $\dot{v}$ is negative semi definite, i.e. $v\left(\tilde{\underline{x}}(t), \hat{\omega}(t), \hat{\underline{\theta}}(t), \hat{\sigma}(t)\right) \leq v\left(\tilde{\underline{x}}(0), \hat{\omega}(0), \hat{\underline{\theta}}(0), \hat{\sigma}(0)\right)$ impose that $\tilde{\underline{x}}^T(t)Pb$, $\hat{\omega}(t)$, $\hat{\underline{\theta}}(t)$, $\hat{\sigma}(t)$ are bounded. Let, $\Psi = \frac{1}{2}\tilde{\underline{x}}^T(t)Q\tilde{\underline{x}}(t) \leq -\dot{v}$ and taking integration of it with respect to time t gives: $\int_0^t \Psi(\tau)d\tau \leq v\left(\tilde{\underline{x}}(0), \hat{\omega}(0), \hat{\underline{\theta}}(0), \hat{\sigma}(0)\right) - v\left(\tilde{\underline{x}}(t), \hat{\omega}(t), \hat{\underline{\theta}}(t), \hat{\sigma}(t)\right)$

As, $v\left(\tilde{\underline{x}}(0), \hat{\omega}(0), \hat{\underline{\theta}}(0), \hat{\sigma}(0)\right)$ is bounded and $v\left(\tilde{\underline{x}}(t), \hat{\omega}(t), \hat{\underline{\theta}}(t), \hat{\sigma}(t)\right)$ is non-increasing and bounded, therefore it follows that, $\lim_{t\to\infty}\int_0^t \Psi(\tau)d\tau < \infty$ and $\dot{\Psi}(t)$ is also bounded. By using Barbalat's lemma we get, $\lim_{t\to\infty}\Psi(t) = 0$ which implying that $\lim_{t\to\infty}\left|\tilde{\underline{x}}\right| = 0$. Thus, $\tilde{\underline{x}}(t)$ is bounded.

3. Main results

3.1. Stochastically optimized hybrid L_1 adaptive controller

It is a very challenging task to properly tune the parameter values of L_1 adaptive controller, as its high adaptive gains lead the system to the verge of instability. During the controller design, L_1 norm condition provides the ranges of the parameter values which does not guaranteed optimal performance of the controller. Thus, the present work utilizes a stochastic optimization technique, such as, harmony search (HS) algorithm to design optimal L_1 adaptive control law to acquire high adaptability with guaranteed stability. HS is employed here due to its low computational cost and robust exploration and exploitation capability. The performance and reliability of L_1 adaptive controller has been improved with this design methodology.

3.2. Harmony search algorithm

Solution of an optimization problem found by a heuristic algorithm is mainly based on trial and error method. Therefore, there is no guarantee of the best or optimal solution. Meta-heuristic algorithm is considered as higher level technique which comprises of lower level technique as well as exploration and exploitation of huge search spaces [31–33]. This can be compared with a musical orchestra where, at the composition period musicians play different combinations of musical pitches randomly. At the practice period they play some music pitches either from their memory or completely new one. When they consider musical pitches from memory then they may adjust or fine tune those pitches to get more correct melody. In this improvisation process their aim is to play a wonderful harmony altogether. Inspired by this phenomenon at first the size of harmony memory (*HM*) matrix is considered. Each pitch of a harmony played by different musicians is a candidate in the candidate solution vector placed at each row of *HM*. One extra column is added in each row that stores the fitness value of each harmony. To generate a new harmony, musicians can choose any of these three processes, (1) may directly take some pitches from the memory, and subsequently, fine tuning of those pitches may be done, (2) can play randomly a new pitch within the range without retrieving from the memory. For first case harmony memory considering rate (*HMCR*), pitch adjust rate (*PAR*) and its bandwidth (*bw*) are required. Depending upon fitness values, harmonies are sorted in ascending/ descending order depending on the minimization/ maximization problem. If the fitness value of the newly generated harmony is better than that of the worst harmony in the memory, then it will replace the worst harmony. If maximum number of iteration is reached or, a specific error criteria is satisfied then, the process will be stopped and the best harmony of the harmony memory matrix will be the optimal solution.

3.3. Variants of harmony search algorithm

Different types of variations are also proposed in basic HS algorithm by many researchers [18,32–34]. The basic HS algorithm utilizes fixed values for each of *HMCR*, *PAR* and *bw*. These three significant parameters are very critical in fine-tuning the optimized candidate solution vectors and can be potentially useful in regulating the convergence rate of the algorithm to accomplish the optimal solution. An improvement of the basic HS algorithm is proposed in [19], where *HMCR* and *PAR* parameters are increased linearly from a minimum value to a maximum value and *bw* is kept constant throughout harmony improvisations. The physical implication of this variant is that, as the generation of harmony improvisation increases, the harmony memory would be rich in experience. The *PAR* parameter is also increased because the fine tuning is required as the optimization algorithm relies more on the past values of the harmonies stored in the memory [19]. Along with this modification of basic HS algorithm, the entire harmony memory had also been improvised [20]. These control parameter variation of HS algorithm to improve the performance and convergence rate of the optimization process are guided by the notional approach of improvising musical instruments in a rehearsal or in concert performance.

Furthermore, along with the earlier modifications in HS algorithm, another parameter, viz. group memory considering rate (*GMCR*) was introduced in [20]. In this variant of HS algorithm, the concept of local neighbourhood topology was introduced where the harmony memory is divided into number of groups and the member harmonies of each group is selected in each iteration. Each harmony belonging to a particular group is improvised either from its own group or from the others according

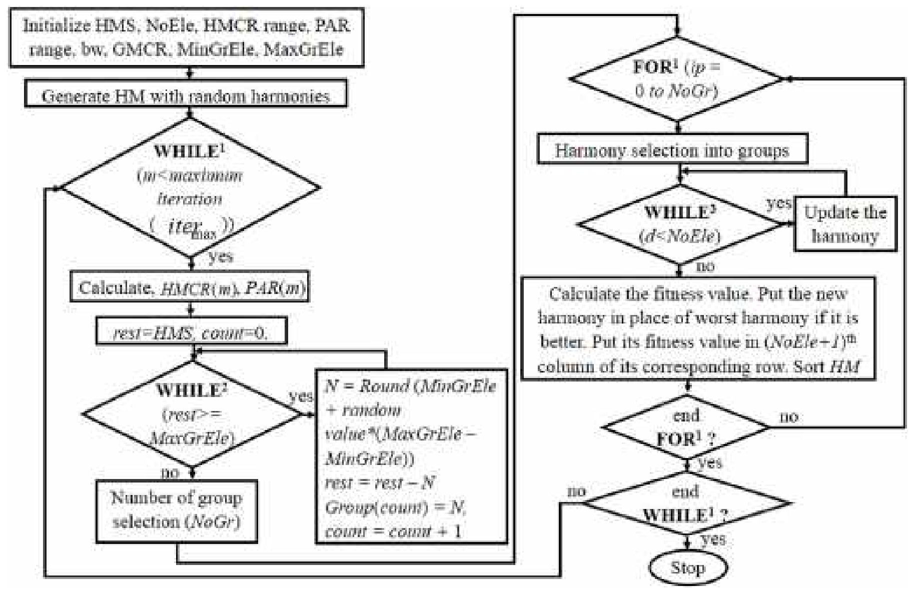

Fig. 2. Flowchart representation of lbest HS algorithm.

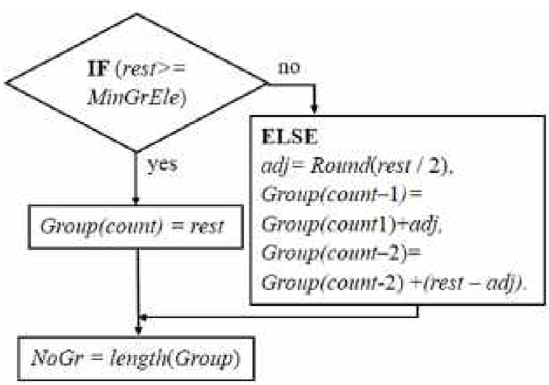

Fig. 3(a). Flowchart for number of group selection.

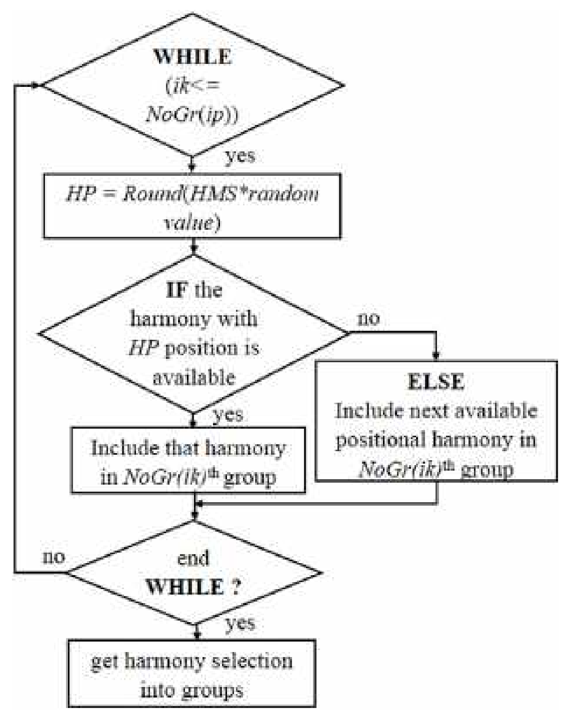

Fig. 3(b). Flowchart for harmony selection into groups.

to the value of $GMCR \in (0, 1)$. The harmony members in each group are reorganized in each iteration, so a more stochastic exploration of the search space could be possible in this lbest variant of HS algorithm. The dynamics of harmony improvisation increases the tendency of achieving the global optimum [20]. This notion of harmony improvisation is conceptualized from the mutual cooperation of different music groups and their combined performance in a philharmonic orchestra. In this work, the basic HS algorithm and this variant have been separately utilized to optimize the L_1 adaptive controller parameters and a comparative study is presented. Fig. 1 describes the lbest version of HS algorithm, termed as LBHS in this paper. The flowchart 1, given in Fig. 2, consists of three more sub-flowcharts, given in Fig. 3.

The number of group selection, harmony selection into groups and update of the harmony, in flowchart 1(Fig. 2) are described by sub-flowcharts, given in Fig. 3(a)–Fig. 3(c).

3.4. Hybrid L_1 adaptive controller

In this hybrid L_1 adaptive controller design process, the Lyapunov theory based L_1 adaptive approach and the lbest version of HS algorithm, namely LBHS, run concurrently to optimize the (i) feedback gain (k), (ii) feed-forward gain of the pre-filter (k_g), (iii) cut-off frequency of the low-pass pre-filter (ρ), (iv) adaptation gain vector ($\underline{\Gamma}$), (v) unknown constant related to system (ω), (vi) parameter uncertainty vector ($\underline{\theta}$) and (vii) time varying disturbance of the system model (σ).

In this method, a harmony vector $\underline{Z}$ can be formed as [32]:

$$\underline{Z} = [k|k_g|\rho|\underline{\Gamma}|\omega|\underline{\theta}|\sigma] \tag{33}$$

The harmony vector $\underline{Z}$ contains all the required information to construct a L_1 adaptive controller. Here, the adaptation gain vector ($\underline{\Gamma}$) can viewed as $\underline{\Gamma} = [\Gamma_1|\Gamma_2|\Gamma_3]$, where Γ_1, Γ_2 and Γ_3 are the adaptation gains or learning rates of ω, $\underline{\theta}$ and σ adaptation respectively. In proposed concurrent hybrid strategy, some of the parameters of the harmony are static and some are varying dynamically in each iteration during the optimization process.

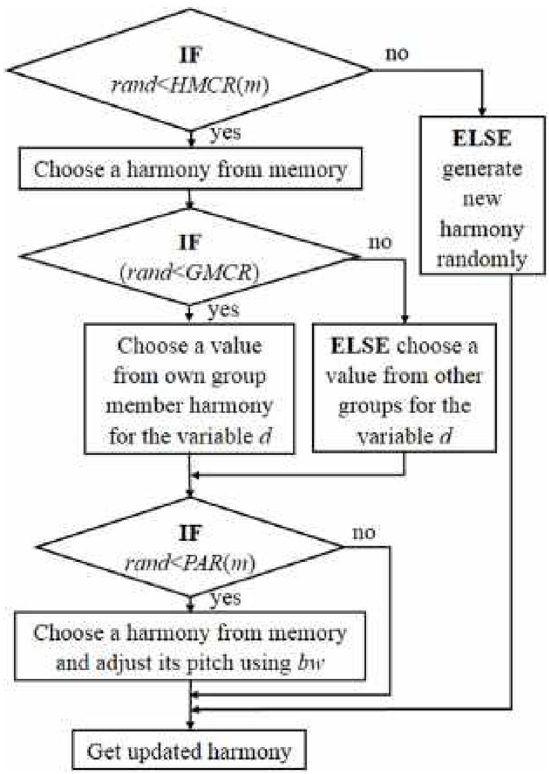

Fig. 3(c). Flowchart for update of harmony.

Thus, the harmony vector $\underline{Z}$ can be split into two parts as:

$$\underline{Z} = [\underline{\psi}|\underline{\mu}] \tag{34}$$

where,

$$\begin{aligned}\text{Static part}: \underline{\psi} = {} & [\text{feedback gain } (k) \mid \text{feed-forward} \\ & \text{gain of the pre-filter } (k_g) \mid \ldots \\ & \mid \text{cut-off frequency of the low-pass pre-filter } (\rho) \mid \\ & \text{adaptation gain vector } (\underline{\Gamma})]\end{aligned} \tag{35}$$

$$\begin{aligned}\text{Dynamic part}: \underline{\mu} = {} & [\text{unknown constant related} \\ & \text{to system } (\omega) \mid \text{parameter} \ldots \\ & \text{uncertainty vector } (\underline{\theta}) \mid \text{time varying disturbance} \\ & \text{of the system model } (\sigma)]\end{aligned} \tag{36}$$

The partition of harmony vector $\underline{Z}$ splits the parameters in such a manner that $\underline{\mu}$ has adaptive effect and $\underline{\psi}$ has non-adaptive effect on $u(s)$ respectively [20,34]. In L_1 adaptive control scheme, the values of the free parameters in the vector $\underline{\psi}$ are defined *a priori* and the value of the corresponding $\underline{\mu}$ minimizing the tracking error can be obtained by the adaptation laws as in (4), (5) and (6) respectively. So, in conventional L_1 adaptive control scheme, the $\underline{\psi}$ is defined by trial and error method while $\underline{\mu}$ will reach its final value by adaptation. In this work, LBHS algorithm is applied to optimize $\underline{\psi}$ and $\underline{\mu}$ in tandem by a method as shown in Figs. 1, 2 to 3. In addition to that, adaptation laws, as in (4), (5) and (6) are applied to improve a new harmony based on LBHS algorithm, to adjust the values of ω, $\underline{\theta}$ and σ only resulting to a new value of $\underline{\mu}$. Concurrent adaptation incurs new harmony vector $\underline{Z_C} = \underline{Z} + \underline{\hat{Z}}$, where adaptation part $\underline{\hat{Z}} = [\underline{\hat{\psi}}|\underline{\hat{\mu}}] = [0|\underline{\hat{\mu}}]$. So, in this methodology, ω, $\underline{\theta}$ and σ explore both the local search space and global search space, simultaneously, in a twofold manner, as described in Fig. 4. The integral absolute error is calculated as $IAE = \int_0^t |r(t) - y(t)|\, dt$ and is taken as fitness function for the optimization problem. Properly tuned harmony vector $\underline{Z_C}$ leads to achieve a bounded control signal to the system along with a stable transient and steady state response.

The architecture of proposed lbest HS based L_1 adaptive controller is given in Fig. 5.

For any multi-agent search algorithm, like HS algorithm, the proper balance between exploration and exploitation is of great importance. The exploration capability depends on the growth of the population variance. If the population variance increases over the iterations then the algorithm has good explorative power to explore new search spaces. Theorem 1 and Lemma 1 explain the explorative capability of the proposed hybrid L_1 adaptive control scheme. At the same time the exploitation competency will be determined by the convergence of the iterative process within its spectral radius. Utilizing Theorem 2 and Lemma 2 the stability, in the sense of convergence, of the entire optimization process for hybrid L_1 adaptive control scheme has been described.

Theorem 1. *Let $Z_C = Z + \hat{Z} = \{Z_{c1}, Z_{c2}, Z_{c3}, \ldots, Z_{cHMS}\}$ be the current population and $Y = \{Y_1, Y_2, Y_3, \ldots, Y_{HMS}\}$ be some intermediate population obtained from new harmony improvisation step of lbest HS Algorithm. With varying HMCR i.e. HMCR(t), the HM consideration probability; varying PAR i.e. PAR(t), the pitch-adjustment probability; fixed bw, the arbitrary distance bandwidth; fixed GMCR, group memory considering rate and the allowable range for the decision variables $(Z, \hat{Z})$ to be $\{Z_{\min}, Z_{\max}\}$ and $\left\{\hat{Z}_{\min}, \hat{Z}_{\max}\right\}$ respectively, where $Z_{\min} = a, Z_{\max} = b, \hat{Z}_{\min} = c, \hat{Z}_{\max} = d$ with $a, b, c, d \in \Re$, and the following equation will hold:*

$$E(Var(Y)) = \left(1 - \frac{1}{HMS}\right) \times \begin{bmatrix} \overline{HMCR} \cdot GMCR \cdot Var(Z_C) \\ + \frac{1}{3} \cdot \overline{HMCR} \cdot GMCR \cdot \overline{PAR} \cdot bw^2 \\ + \frac{1}{2} \cdot (1 - \overline{HMCR} \cdot GMCR)(a+b)(c+d) \\ + \frac{1}{3} \cdot (1 - \overline{HMCR} \cdot GMCR)(a^2 + a \cdot b + b^2 + c^2 + cd + d^2) \\ + \overline{HMCR} \cdot GMCR \cdot (1 - \overline{HMCR} \cdot GMCR) \cdot \overline{Z_C}^2 \\ - \frac{1}{4} \cdot (1 - \overline{HMCR} \cdot GMCR)^2 \cdot (a+b+c+d)^2 \\ - \overline{HMCR} \cdot GMCR \cdot (1 - \overline{HMCR} \cdot GMCR) \cdot \overline{Z_C}(a+b+c+d) \end{bmatrix} \tag{37}$$

Proof. If $Z_C = \{Z_{c1}, Z_{c2}, Z_{c3}, \ldots, Z_{cHMS}\}$ is the current population, then the population mean and quadratic population mean is given by $\overline{Z_C} = (1/HMS) \sum_{i=1}^{HMS} Z_{ci}$ and $\overline{Z_C^2} = (1/HMS) \sum_{i=1}^{HMS} Z_{ci}^2$ respectively. $E(Var(Z_C))$ is the explorative power measurement of the population variance $Var(Z_C) = \frac{1}{HMS} \sum_{i=1}^{HMS} \left(Z_{ci} - \overline{Z_C}\right)^2 = \overline{Z_C^2} - \overline{Z_C}^2$. Let Y with each element Y_i is an intermediate population of harmony memory improvisation process. Then for concurrent lbest version of HS, each element Y_i can be expressed as:

$$Y_i = \begin{cases} Z_r + \hat{Z}_r, & \text{with probality } HMCR(t) \cdot GMCR \cdot (1 - PAR(t)) \\ Z_r + \hat{Z}_r + bw \cdot rand, & \text{with probality } 0.5HMCR(t) \cdot GMCR \cdot PAR(t) \\ Z_r + \hat{Z}_r - bw \cdot rand, & \text{with probality } 0.5HMCR(t) \cdot GMCR \cdot PAR(t) \\ Z_{new} + \hat{Z}_r, & \text{with probality } (1 - HMCR(t) \cdot GMCR) \end{cases}$$

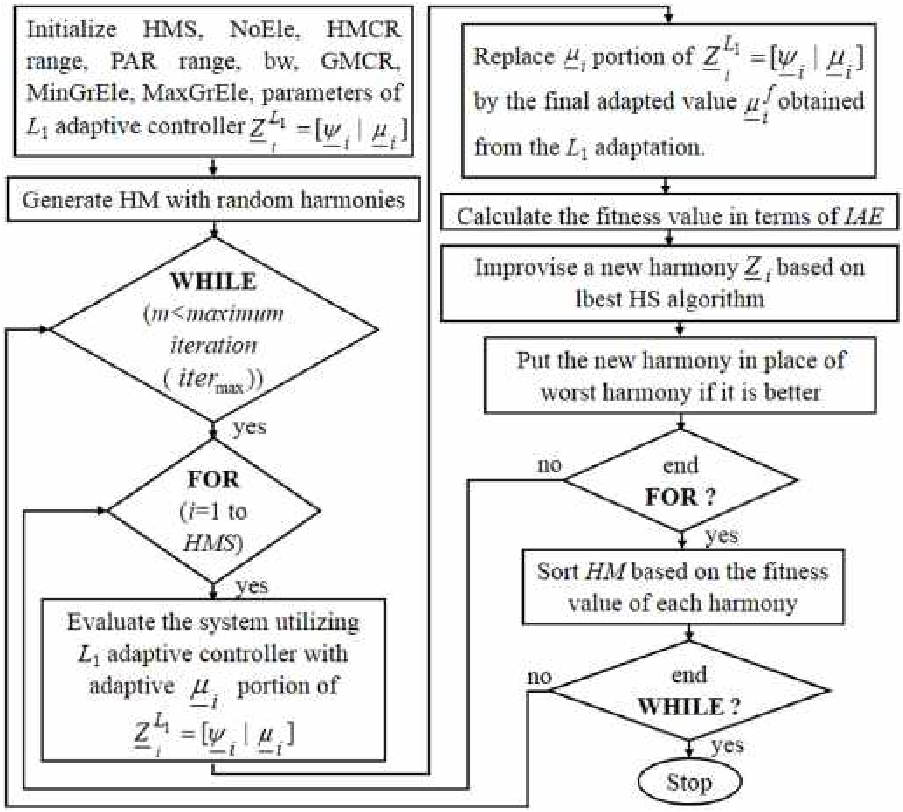

Fig. 4. Flowchart representation of lbest HS based L_1 adaptive controller.

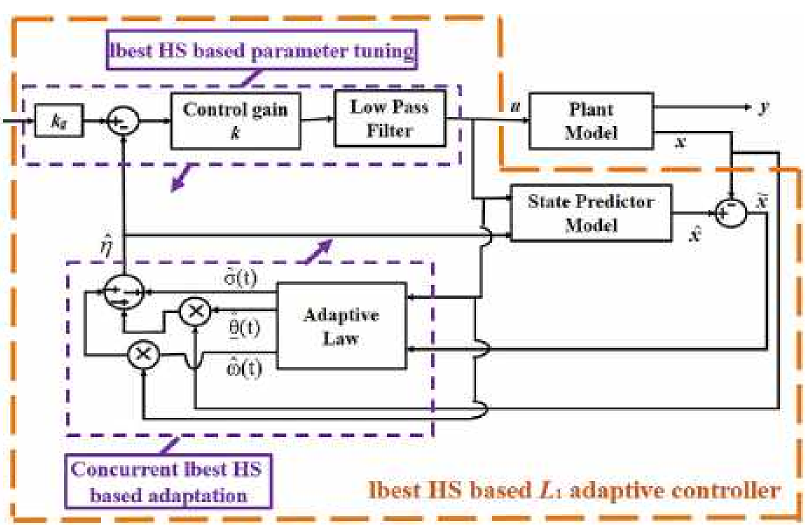

Fig. 5. Architecture of proposed lbest HS based L_1 adaptive controller.

Assuming, $Z_{C_r} = Z_r + \hat{Z}_r$ is a random variable with uniformly distributed random index $r \in \{1, 2, 3, \ldots, HMS\}$, with probability $p_k = P(r = k) = (1/HMS)$, $k \in \{1, 2, 3, \ldots, HMS\}$.

The expectation of Z_{C_r} and $Z_{C_r}^2$ can be given as [35]:

$$E(Z_{C_r}) = \sum_{k=1}^{HMS} p_k \cdot Z_{C_k} = \frac{1}{HMS}\sum_{k=1}^{HMS} Z_{C_k} = \overline{Z_C}$$

$$\text{and } E(Z_{C_r}^2) = \sum_{k=1}^{HMS} p_k \cdot Z_{C_k}^2 = \frac{1}{HMS}\sum_{k=1}^{HMS} Z_{C_k}^2 = \overline{Z_C^2}$$

Expectation of Y_i is given as:

$$\begin{aligned} E(Y_i) &= HMCR(t) \cdot GMCR \cdot (1 - PAR(t)) \cdot E(Z_{C_r}) \\ &+ 0.5 \cdot HMCR(t) \cdot GMCR \cdot PAR(t) \cdot E(Z_{C_r} + bw \cdot rand) \\ &+ 0.5 \cdot HMCR(t) \cdot GMCR \cdot PAR(t) \cdot E(Z_{C_r} - bw \cdot rand) \\ &+ (1 - HMCR \cdot GMCR) \cdot E(Z_{new} + \hat{Z}_r) \end{aligned} \tag{38}$$

Therefore, expectation of Y_i^2 is given as:

$$\begin{aligned} E(Y_i^2) &= HMCR(t) \cdot GMCR \cdot (1 - PAR(t)) \cdot E(Z_{C_r}^2) \\ &+ 0.5 \cdot HMCR(t) \cdot GMCR \cdot PAR(t) \cdot E(Z_{C_r}^2 + bw \cdot rand)^2 \\ &+ 0.5 \cdot HMCR(t) \cdot GMCR \cdot PAR(t) \cdot E(Z_{C_r}^2 - bw \cdot rand)^2 \\ &+ (1 - HMCR(t) \cdot GMCR) \cdot E(Z_{new} + \hat{Z}_r)^2 \end{aligned} \tag{39}$$

Now, the values of $Z_{new}, Z_{new}^2, \hat{Z}_r, \hat{Z}_r^2$ have to be found out.Z_{new} is a randomly selected variable between the range $Z_{\min} = a, Z_{\max} = b$, following the formula: $Z_{new} = Z_{\min} + rand \cdot (Z_{\max} - Z_{\min})$. $\hat{Z}_r$ is a randomly selected variable between the range $\hat{Z}_{r\,\min} = c, \hat{Z}_{r\,\max} = d$. Where, rand is a random number between 0 and 1 satisfying a continuous uniform probability density function:

$$\alpha\,(rand) = \begin{cases} 1, & rand \in [0, 1] \\ 0, & otherwise \end{cases}$$

Now,

$$E(rand) = \int_0^1 rand \cdot \alpha(rand) \cdot drand = \left[\frac{rand^2}{2}\right]_0^1 = \frac{1}{2}$$

$$E(rand^2) = \int_0^1 rand^2 \cdot \alpha(rand) \cdot drand = \left[\frac{rand^3}{3}\right]_0^1 = \frac{1}{3}$$

Therefore,

$$E(Z_{new}) = a + E(rand) \cdot (b - a) = \frac{a+b}{2}$$

$$E\left((Z_{new})^2\right) = a^2 + 2 \cdot a \cdot E(rand) \cdot (b-a) + E(rand^2) \cdot (b-a)^2 = \frac{a^2 + ab + b^2}{3}$$

Similarly,

$$E(\hat{Z}_r) = \frac{c+d}{2}$$

$$E\left(\left(\hat{Z}_r\right)^2\right) = \frac{c^2 + cd + d^2}{3}$$

By putting the values of $E(Z_{new})$ and $E\left((Z_{new})^2\right)$ in the Eqs. (38) and (39) we get,

$$\begin{aligned} E(Y_i) &= HMCR(t) \cdot GMCR \cdot E(Z_{C_r}) \\ &\quad + \frac{1}{2}(1 - HMCR(t) \cdot GMCR)(a+b+c+d) \\ &= HMCR(t) \cdot GMCR \cdot \overline{Z_C} + \frac{1}{2}(1 - HMCR(t) \cdot GMCR)(a+b+c+d) \end{aligned} \tag{40}$$

and

$$\begin{aligned} E\left((Y_i)^2\right) &= HMCR(t) \cdot GMCR \cdot E\left((Z_{C_r})^2\right) \\ &+ \frac{bw^2}{3} HMCR(t) \cdot GMCR \cdot PAR(t) \\ &+ \frac{1}{2} \cdot (1 - HMCR(t) \cdot GMCR)(a+b)(c+d) \\ &+ \frac{1}{3}(1 - HMCR(t) \cdot GMCR)(a^2 + ab + b^2 + c^2 + cd + d^2) \\ &= HMCR(t) \cdot GMCR \cdot \overline{Z_C^2} + \frac{bw^2}{3} HMCR(t) \cdot GMCR \cdot PAR(t) \\ &+ \frac{1}{2} \cdot (1 - HMCR(t) \cdot GMCR)(a+b)(c+d) \\ &+ \frac{1}{3}(1 - HMCR(t) \cdot GMCR)(a^2 + ab + b^2 + c^2 + cd + d^2) \end{aligned} \tag{41}$$

As mean of Y is given by: $\overline{Y} = \frac{1}{HMS} \sum_{i=1}^{HMS} Y_i$

The expectation of $\overline{Y}$ is given by:

$$\begin{aligned} E\left(\overline{Y}\right) &= \frac{1}{HMS} \sum_{i=1}^{HMS} E\left(Y_i\right) \\ &= \frac{1}{HMS} \sum_{i=1}^{HMS} \left[\begin{array}{l} HMCR(t) \cdot GMCR \cdot \overline{Z_C} \\ + \frac{1}{2}(1 - HMCR(t) \cdot GMCR)(a+b+c+d) \end{array}\right] \\ &= \overline{HMCR} \cdot GMCR \cdot E(\overline{Z_C}) + \frac{1}{2}(1 - \overline{HMCR} \cdot GMCR)(a+b+c+d) \end{aligned} \tag{42}$$

where, $\frac{1}{HMS} \sum_{i=1}^{HMS} HMCR(t) = \overline{HMCR}$

Similarly, $\overline{Y^2} = \frac{1}{HMS} \sum_{i=1}^{HMS} (Y_i)^2$

The expectation of $\overline{Y^2}$ is given by:

$$\begin{aligned} E\left(\overline{Y^2}\right) &= \frac{1}{HMS} \sum_{i=1}^{HMS} E\left((Y_i)^2\right) \\ &= \frac{1}{HMS} \sum_{i=1}^{HMS} \left[\begin{array}{l} HMCR(t) \cdot GMCR \cdot \overline{Z_C^2} + \frac{bw^2}{3} HMCR(t) \cdot GMCR \cdot PAR(t) \\ + \frac{1}{2} \cdot (1 - HMCR(t) \cdot GMCR)(a+b)(c+d) \\ + \frac{1}{3}(1 - HMCR(t) \cdot GMCR)(a^2 + ab + b^2 + c^2 + cd + d^2) \end{array}\right] \\ &= \overline{HMCR} \cdot GMCR \cdot \overline{Z_C^2} + \frac{bw^2}{3} \cdot \overline{HMCR} \cdot GMCR \cdot \overline{PAR} \\ &+ \frac{1}{2} \cdot (1 - \overline{HMCR} \cdot GMCR)(a+b)(c+d) \\ &+ \frac{1}{3}(1 - \overline{HMCR} \cdot GMCR)(a^2 + ab + b^2 + c^2 + cd + d^2) \end{aligned} \tag{43}$$

where, $\frac{1}{HMS} \sum_{i=1}^{HMS} HMCR(t) = \overline{HMCR}$ and $\frac{1}{HMS} \sum_{i=1}^{HMS} PAR(t) = \overline{PAR}$ and, $\overline{Y}^2 = \left(\frac{1}{HMS}\right)^2 \left(\sum_{i=1}^{HMS} (Y_i)\right)^2$

$$\begin{aligned} E\left(\overline{Y}^2\right) &= \left(\frac{1}{HMS}\right)^2 \left(\sum_{i=1}^{HMS} (Y_i)\right)^2 \\ &= \frac{1}{HMS} \cdot E\left[\frac{1}{HMS} \cdot \sum_{i=1}^{HMS} (Y_i)^2\right] \\ &+ \frac{(HMS-1)}{HMS} \cdot E\left[\frac{1}{HMS} \cdot \sum_{\substack{i,j=1 \\ i \neq j}}^{HMS} Y_i \cdot Y_j\right] \end{aligned}$$

As, $E\left(Y_i \cdot Y_j\right) = E\left(Y_i\right) \cdot E\left(Y_j\right)$, therefore,

$$E\left(\overline{Y}^2\right) = \frac{1}{HMS} \cdot E\left(\overline{Y^2}\right) + \frac{(HMS-1)}{HMS} \cdot \left[E\left(\overline{Y}\right)\right]^2 \tag{44}$$

Now,

$$\begin{aligned} E(Var(Y)) &= E\left(\frac{1}{HMS} \sum_{i=1}^{HMS} (Y_i - \overline{Y})^2\right) = E\left(\overline{Y^2}\right) - E\left(\overline{Y}^2\right) \\ &= \frac{HMS-1}{HMS} \cdot \left[E\left(\overline{Y^2}\right) - \left[E\left(\overline{Y}\right)\right]^2\right] \end{aligned} \tag{45}$$

Putting the values from Eqs. (42) and (43), in Eq. (45), one will get:

$$E(Var(Y)) = \left(1 - \frac{1}{HMS}\right)$$

$$\times\begin{bmatrix}\overline{HMCR}\cdot GMCR\cdot\overline{Z_C^2}+\frac{1}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\cdot bw^2\\+\frac{1}{2}\cdot(1-\overline{HMCR}\cdot GMCR)(a+b)(c+d)\\+\frac{1}{3}\cdot(1-\overline{HMCR}\cdot GMCR)(a^2+a\cdot b+b^2+c^2+cd+d^2)\\-\begin{pmatrix}\overline{HMCR}\cdot GMCR\cdot\overline{Z_C}\\+\frac{1}{2}\cdot(1-\overline{HMCR}\cdot GMCR)\cdot(a+b+c+d)\end{pmatrix}^2\end{bmatrix}$$

or,

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\times\begin{bmatrix}\overline{HMCR}\cdot GMCR\cdot Var(Z_C)+\overline{HMCR}\cdot GMCR\cdot\overline{Z_C}^2\\+\frac{1}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\cdot bw^2\\+\frac{1}{2}\cdot(1-\overline{HMCR}\cdot GMCR)(a+b)(c+d)\\+\frac{1}{3}\cdot(1-\overline{HMCR}\cdot GMCR)(a^2+a\cdot b+b^2+c^2+cd+d^2)\\-\overline{HMCR}^2\cdot GMCR^2\cdot\overline{Z_C}^2\\-\frac{1}{4}\cdot(1-\overline{HMCR}\cdot GMCR)^2\cdot(a+b+c+d)^2\\-\overline{HMCR}\cdot GMCR\cdot(1-\overline{HMCR}\cdot GMCR)\cdot\overline{Z_C}(a+b+c+d)\end{bmatrix}$$

or,

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\times\begin{bmatrix}\overline{HMCR}\cdot GMCR\cdot Var(Z_C)\\+\frac{1}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\cdot bw^2\\+\frac{1}{2}\cdot(1-\overline{HMCR}\cdot GMCR)(a+b)(c+d)\\+\frac{1}{3}\cdot(1-\overline{HMCR}\cdot GMCR)(a^2+a\cdot b+b^2+c^2+cd+d^2)\\+\overline{HMCR}\cdot GMCR\cdot(1-\overline{HMCR}\cdot GMCR)\cdot\overline{Z_C}^2\\-\frac{1}{4}\cdot(1-\overline{HMCR}\cdot GMCR)^2\cdot(a+b+c+d)^2\\-\overline{HMCR}\cdot GMCR\cdot(1-\overline{HMCR}\cdot GMCR)\cdot\overline{Z_C}(a+b+c+d)\end{bmatrix}$$

The theorem is proved. ■

Depending on this theorem, the following lemma can be given to show the exploration capability of this algorithm analytically.

Lemma 1. *If HMCR (t) and GMCR is chosen nearly equal to 1, and the bandwidth bw is chosen to vary depending upon $Var(Z)$ as, $bw=\kappa\sqrt{Var(Z)}$, then with the increase of the iteration the explorative power of the lbest HS algorithm will increase exponentially.*

Proof. If $HMCR(t)\approx 1$ and $GMCR\approx 1$, then $\overline{HMCR}\cdot GMCR\approx 1$ and all the terms related with $(1-\overline{HMCR}\cdot GMCR)$ can be neglected. Then the expectation of population variance becomes:

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\begin{pmatrix}\overline{HMCR}\cdot GMCR\\+\frac{\kappa^2}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\end{pmatrix}\cdot Var(Z_C)$$

or,

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\cdot\overline{HMCR}\cdot GMCR\cdot\left(1+\frac{\kappa^2}{3}\cdot\overline{PAR}\right)\cdot Var(Z_C) \quad (46)$$

After itr number of iteration it become,

$$E(Var(Y_{itr}))=\left\{\left(1-\frac{1}{HMS}\right)\cdot\overline{HMCR}\cdot GMCR\cdot\left(1+\frac{\kappa^2}{3}\cdot\overline{PAR}\right)\right\}^{itr}\cdot Var(Z_C) \quad (47)$$

From Eq. (47), it is evident that the population variance is increasing exponentially over the iterations. Therefore, this algorithm shows exploration capability. This proves the lemma. ■

With the exploration capability it is also important to have good exploitation capability of any searching algorithm so that it can converge at the end. To show the convergence of the proposed concurrent lbest HS based L_1 adaptive, Theorem 2 and Lemma 2 are given below.

Theorem 2. *The iterative equation of lbest version of HS comprises of expectation of population variance $E(Var(Z_C))$ and expectation of population mean $E(\overline{Z_C})$ will converge if the spectral radius of that iterative matrix is less than unity.*

Proof. As, $HMCR\approx 1$, ∴ $\overline{HMCR}\approx 1$, so the term $(1-\overline{HMCR}\cdot GMCR)$ can be omitted from the equations. Then Eqs. (37) and (40) becomes:

$$E\left(\overline{Y}\right)=\overline{HMCR}\cdot GMCR\cdot E(\overline{Z_C}) \quad (48)$$

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\begin{pmatrix}\overline{HMCR}\cdot GMCR\cdot Var(Z_C)\\+\frac{1}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\cdot bw^2\end{pmatrix} \quad (49)$$

Assumption 1. Let us assume, that the bandwidth is proportional with the square root of the expectation mean of population (i.e. $bw\propto\sqrt{E(\overline{Z_C})}=\kappa\sqrt{E(\overline{Z_C})}$).

Therefore,

$$E(Var(Y))=\left(1-\frac{1}{HMS}\right)\begin{pmatrix}\overline{HMCR}\cdot GMCR\cdot Var(Z_C)\\+\frac{\kappa^2}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}\cdot E(\overline{Z_C})\end{pmatrix}$$

Now, the iterative equation becomes:

$$\begin{bmatrix}E(Var(Y))\\E(\overline{Y})\end{bmatrix}=\begin{bmatrix}M_1 & M_2\\M_3 & M_4\end{bmatrix}\cdot\begin{bmatrix}E(Var(Z_C))\\E(\overline{Z_C})\end{bmatrix} \quad (50)$$

where, $M_1=\left(1-\frac{1}{HMS}\right)\cdot\overline{HMCR}\cdot GMCR$,
$M_2=\left(1-\frac{1}{HMS}\right)\cdot\frac{\kappa^2}{3}\cdot\overline{HMCR}\cdot GMCR\cdot\overline{PAR}$,
$M_3=0$,
$M_4=\overline{HMCR}\cdot GMCR$.

It takes a form of iterative equation $Y=M\cdot Z_C$.

where, $Y=\begin{bmatrix}E(Var(Y))\\E(\overline{Y})\end{bmatrix}$, $Z_C=\begin{bmatrix}E(Var(Z_C))\\E(\overline{Z_C})\end{bmatrix}$ and $M=\begin{bmatrix}M_1 & M_2\\M_3 & M_4\end{bmatrix}$.

The condition of convergence for this iterative matrix M depends on this following lemma.

Lemma 2. *If the spectral radius of any iterative matrix M is $\rho(M)$, then, the iterative matrix will converge if $\rho(M)<1$. Mathematically, it can be written as:*

$$\text{if} \quad \rho(M)<1 \text{ then } \lim_{n\to\infty}\left\|M^n\right\|=0 \quad (51)$$

$$\text{and if} \quad \rho(M)>1 \text{ then } \lim_{n\to\infty}\left\|M^n\right\|=\infty \quad (52)$$

Proof. Proof for this lemma is given in Appendix. ■

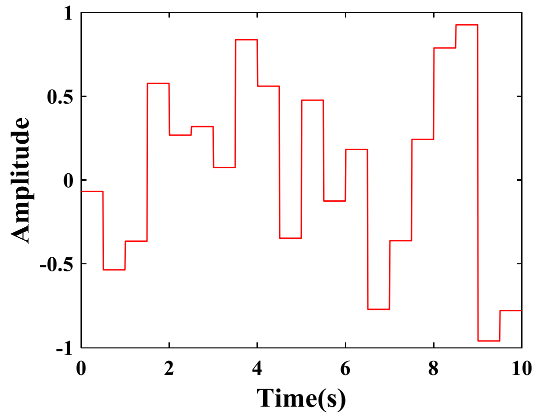

Fig. 6. Nature of the disturbance.

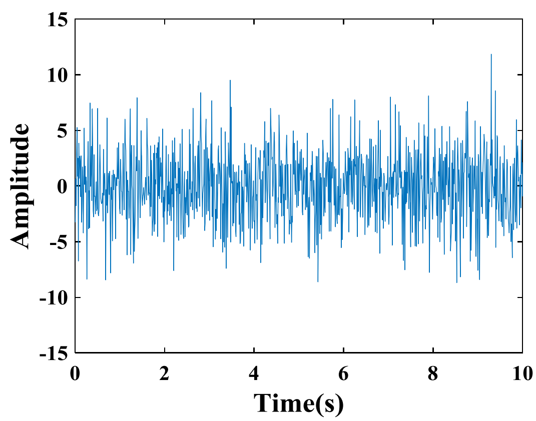

Fig. 7. Nature of 10 dbw Gaussian noise.

The characteristic roots of iterative matrix given in Eq. (50) is,

$$\lambda_1 = \left(1 - \frac{1}{HMS}\right)\overline{HMCR} \cdot GMCR \text{ and } \lambda_2 = \overline{HMCR} \cdot GMCR.$$

Now, as $HMS > 1, \therefore \left(1 - \frac{1}{HMS}\right) < 1$, $HMCR(t) < 1, \therefore \overline{HMCR} < 1$ and $GMCR < 1$, therefore $\lambda_1 < 1$.

From this observation, the second root $\lambda_2 < 1$.

Again, $\frac{\lambda_1}{\lambda_2} = \left(1 - \frac{1}{HMS}\right)$. As, $HMS > 1, \therefore \left(1 - \frac{1}{HMS}\right) < 1$, so, $\lambda_2 > \lambda_1$.

Therefore, the spectral radius of iterative matrix M is $\rho(M) = MAX\,(\lambda_1, \lambda_2) = \lambda_2 < 1$.

Thus the iteration equation of harmony search algorithm will converge.

It completes the proof. ■

In case of conventional HS algorithm, the spectral radius of iterative matrix (M_{conv}) can be given as [36] $\rho(M_{conv}) = \underline{HMCR}$. Whereas, in this case for lbest HS algorithm $\underline{\rho(M)} = \overline{HMCR} \cdot \underline{GMCR}$. It is evident that, $\underline{\rho(M)} < \rho(M_{conv})$, as, $\overline{HMCR} \approx HMCR$, $\overline{HMCR} < 1, GMCR < 1$; $\overline{HMCR} \cdot GMCR < HMCR$. Therefore the rate of convergence of the proposed lbest variant of HS algorithm, i.e. LBHS is higher than that of the conventional HS algorithm.

From these analysis, it can be clearly shown that the LBHS based concurrent hybrid L_1 adaptive controller is efficient enough to give better convergence which incurs good transient performance as well as high robustness.

4. Simulation and experimental case studies

To demonstrate the effectiveness of the proposed LBHS based concurrent hybrid L_1 adaptive control scheme for tracking control of a class of non-linear systems both simulation and experimental studies are performed. Benchmark systems, like Duffing's oscillatory chaotic system, spring–mass–damper system, 4th order unstable non-minimum phase cart–pendulum system are considered in simulation case studies, whereas in experimental case study speed control of a DC motor with uncertainty is studied. The non-linear dynamics of the systems are simulated using a fixed step 4th order Runge–Kutta method, with step size (sampling time) $\Delta t = 0.01$ s for chaotic Duffing's system, spring–mass–damper system, cart–pendulum system and $\Delta t = 0.1$ s for DC motor. During L_1 adaptive controller operation at first the parameters are adapted for 200 s and then, the adapted controller is evaluated for 10 s. The adaptation gains ($\underline{\Gamma}$) are switched to zero during the evaluation period. In HS algorithm based designs, 30 numbers of harmonies are taken and 200 harmony improvisations are performed to find the best candidate controller. The stochastic optimization techniques for obtaining best candidate controller run 10 times each and then the optimal candidate controllers evaluate the system for 10 s duration. For each case studies, the results of the proposed hybrid local best HS (HybdLBHS) based L_1 adaptive control method is compared with different types of control strategies viz. local best harmony search (LBHS) based state feedback control, basic L_1 adaptive control, PID augmented L_1 adaptive control, fuzzy feedback filter based L_1 Adaptive control, hybrid PSO (HybdPSO) based L_1 adaptive control, hybrid HS (HybdHS) based L_1 adaptive control. The description of these control methodologies are briefed below.

LBHS based state feedback tracking control: In this scheme, the control law is formulated as the conventional state feedback tracking control, without using L_1 adaptive controller, and the parameter values are tuned by using lbest HS algorithm. 30 numbers of harmonies are improvised for 200 times to get optimal setting of the controller parameters.

L_1 **adaptive control** [10,11,21]**:** In this design strategy, only unknown constant related to system (ω), parameter uncertainty vector ($\underline{\theta}$), time varying disturbance of the system model (σ) are adapted and the static $\underline{\psi}$ is manually set a priori. The adaptation starts at t = 0s and continues up to t = 200 s. Then, the system is evaluated for 10 s with this reference signal. During this period, adaptation is turned off and the controller operates with already adapted parameters.

PID augmented L_1 **adaptive control** [22]**:** In this design technique, conventional L_1 adaptive controller is augmented with PID controller to eliminate the steady state error.

Fuzzy feedback filter based L_1 **adaptive control** [23]**:** In this design, the gain to the controller, k, is designed by using fuzzy logic and the fuzzy system is tuned by using particle swarm optimization (PSO).

HybdPSO based L_1 **adaptive control**: Furthermore, particle swarm optimization (PSO) [37], most popularly used in engineering applications, is also utilized to design the hybrid controller in conjunction with L_1 adaptive control strategy and the results are compared to that of the HS based hybrid design technique.

HybdHS based L_1 **adaptive control:** In this technique, basic HS algorithm based optimization of parameters for $Z = [\psi | \mu]$ and L_1 adaptive control strategy, which only adjusts μ, run concurrently. To perform this simulation, population size of 30 harmony is taken. For each simulation 200 harmony improvisation of HS algorithm is set. Here, the adaptation gains and other free parameters of design are optimized by the HS algorithm, thus reducing the error due to manual tuning.

HybdLBHS based L_1 **adaptive control:** In this proposed design technique, only the basic HS algorithm is replaced by the lbest topological model of HS algorithm. All other parameters and settings are kept identical to previous design techniques.

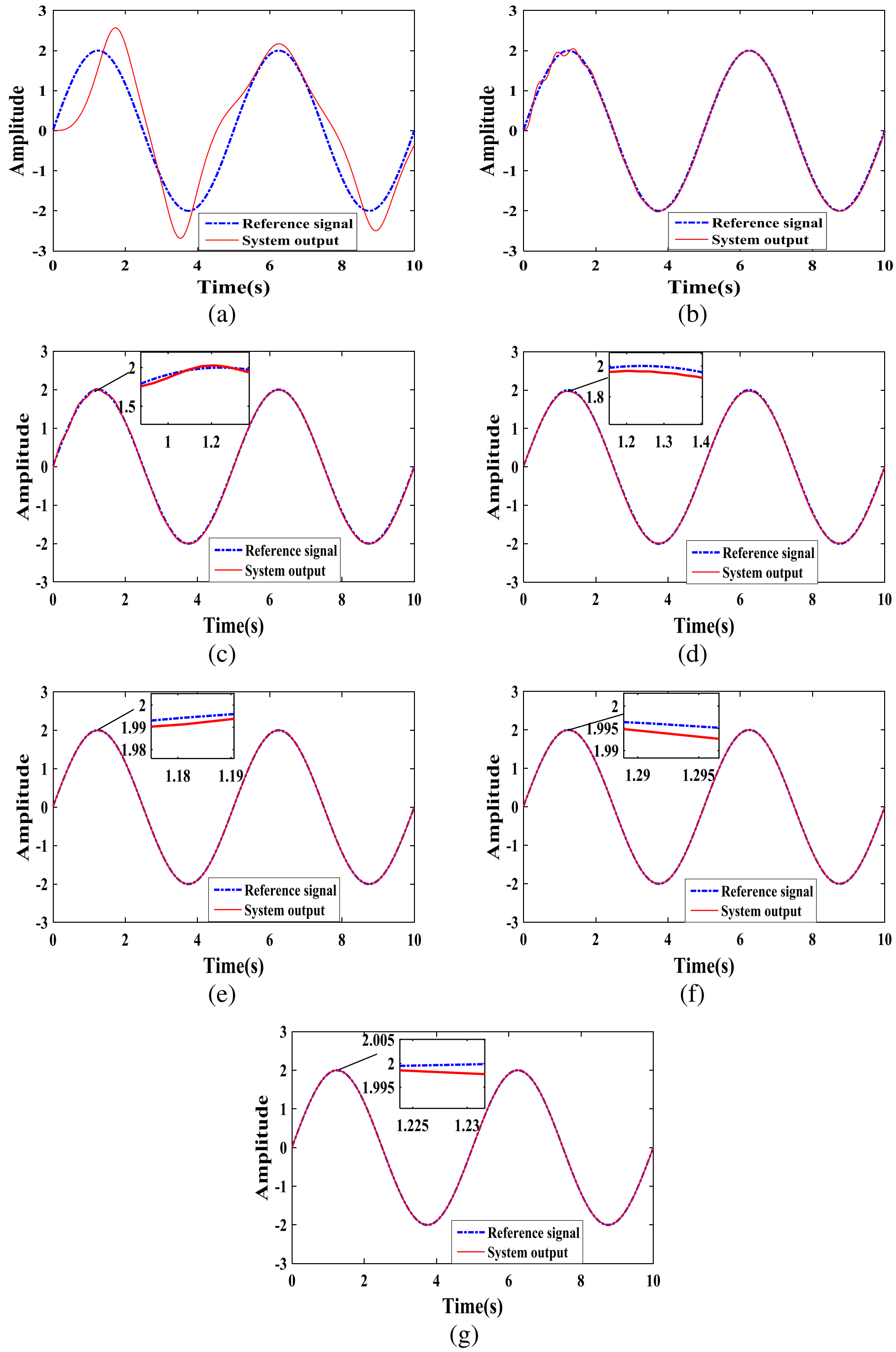

Fig. 8. Evaluation period system response of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study- I.

Case study- I

In this case study, forced oscillatory Duffing's system with disturbance is considered and is given as:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12\cos(t) + u(t) + d(t) \\ y &= x_1 \end{aligned}\right\} \tag{53}$$

where, $d(t)$ represents time varying disturbance with random amplitude varying between $[-1\ 1]$ with a time period of 0.5 s, as given in Fig. 6. The control objective is to track the reference trajectory $y_m = 2 * \sin t$. At the outset, the controllers are tuned for the system with disturbance employing different control strategies. Then, the tuned controllers are also subjected to 10 dbw Gaussian noise, as shown in Fig. 7, to study the robustness of the design procedures. As a good trade-off between adaptation rate and robustness of operation [10,11], the value of adaptation gains for basic L_1 adaptive controller are selected as $\underline{\Gamma} = 9{,}000{,}000\ 9{,}200{,}000\ 8{,}950{,}000$. The results are given in Table 1 and Figs. 8 to 9 show the responses with disturbance and Fig. 10 shows the responses with noise. The control input $u(t)$ is force and the output y is displacement, all are in S.I. unit.

In the first control scheme i.e. LBHS based state feedback tracking control, the approximation of time varying disturbances and uncertainties are not up to the mark and thus, producing unsatisfactory transient performance as depicted in Fig. 8(a). Conventional L_1 adaptive controller arbitrarily chooses its parameters

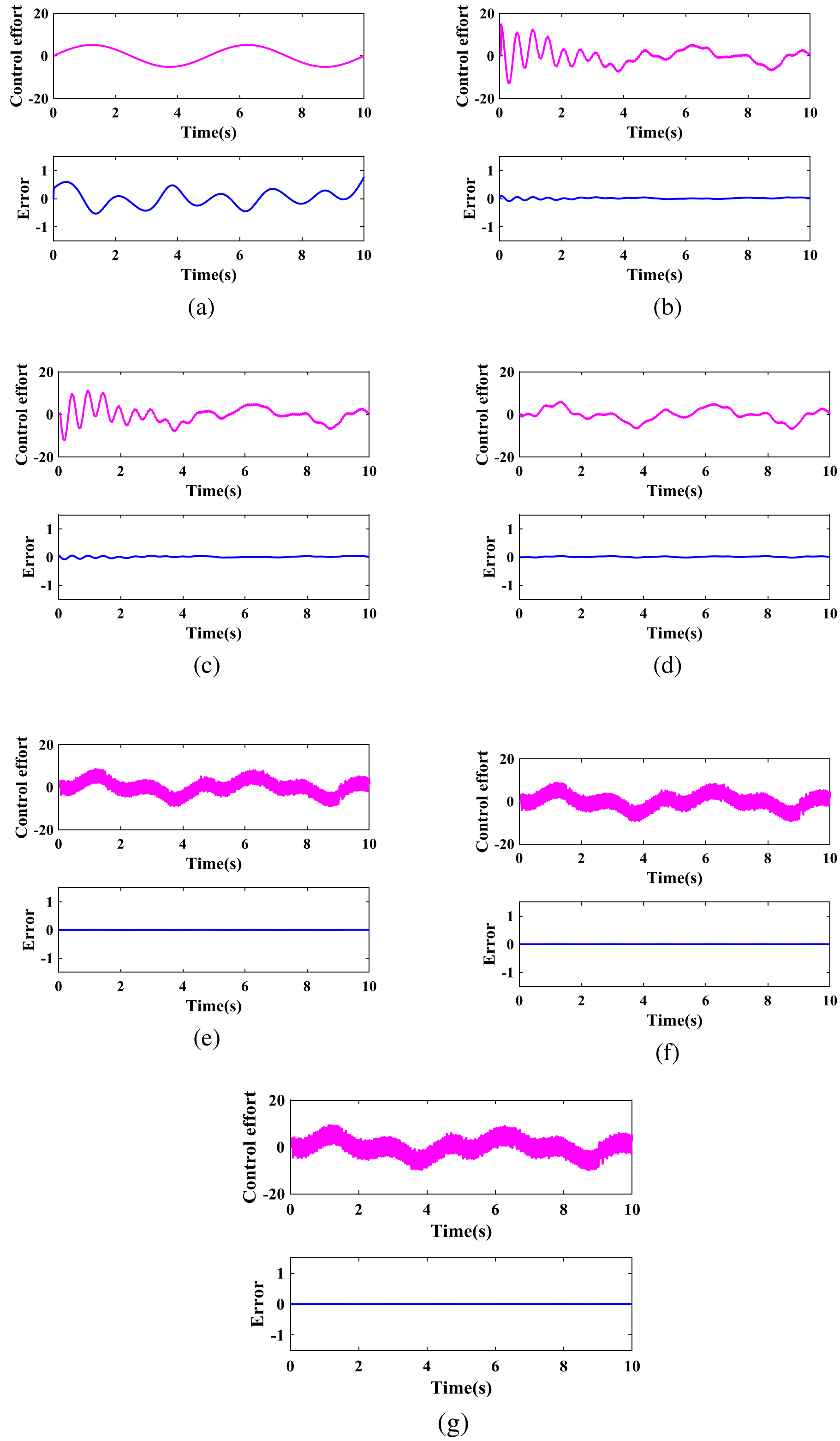

Fig. 9. Evaluation period control effort and error signal of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study- I.

within the range obtained from mathematical calculation of L_1 norm condition [10,11,21], which does not provide optimal result. On the other hand, in the proposed method, the optimal parameter values are evolved by using lbest HS algorithm, which delivers accurate control effort. Therefore without augmenting another controlling technique like PID, the proposed method is capable of giving better performance. Augmenting PID controller with L_1 adaptive controller [22] requires more computational time which is disadvantageous in real-time experimentation. Although, a PSO based fuzzy logic technique [23] is used to tune the gain of the controller k, while other parameters of L_1 adaptive controller are arbitrarily chosen within the ranges obtained from

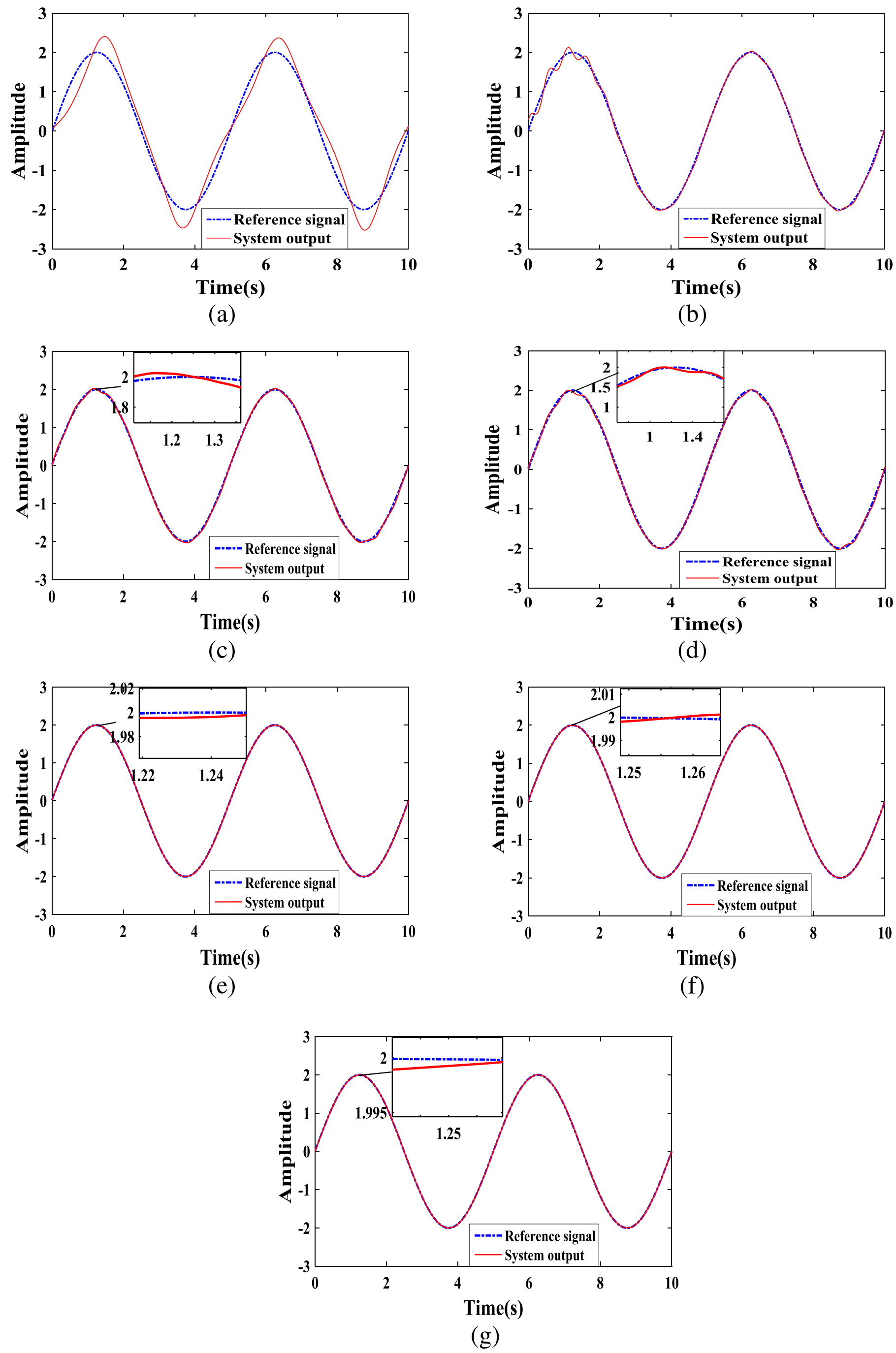

Fig. 10. Evaluation period system response of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study- I with 10 dbw Gaussian noise.

Table 1
Comparative study of different control strategies for case study– I.

Control strategy	IAE value			
	For disturbance			For 10 dbw noise
	Best	Average	Std. Dev.	
LBHS based state feedback tracking	2.3600	2.6115	0.2204	2.6818
L_1 adaptive [10,11,21]	0.2347	–	–	0.4559
PID augmented L_1 adaptive [22]	0.2179	–	–	0.2446
Fuzzy feedback filter L_1 adaptive [23]	0.1645	0.1955	0.0322	0.2854
HybdPSO based L_1 adaptive	0.0129	0.0454	0.0208	0.0199
HybdHS based L_1 adaptive	0.0121	0.0279	0.0128	0.0197
Proposed HybdLBHS based L_1 adaptive	0.0118	0.0119	0.0003	0.0186

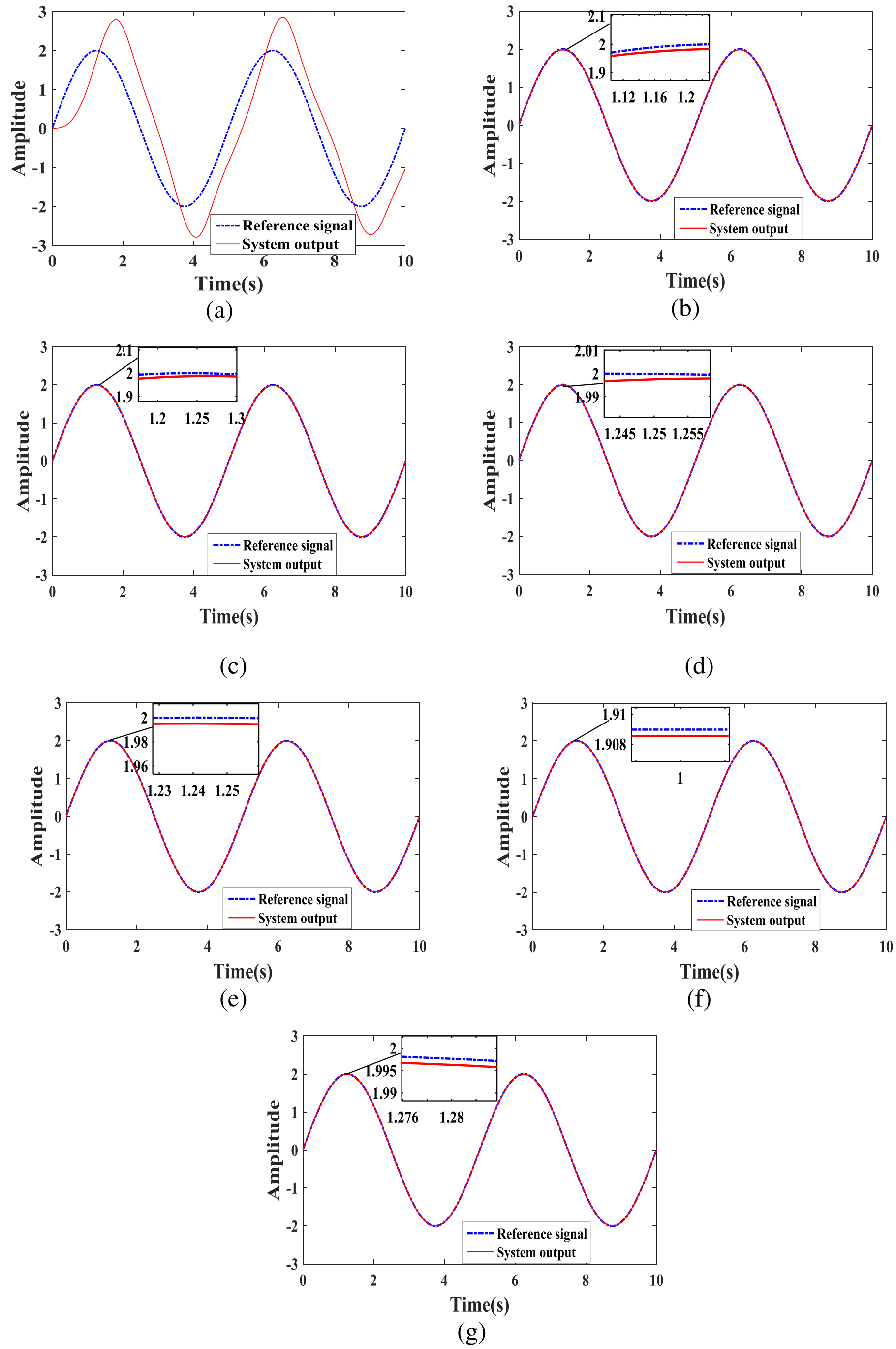

Fig. 11. Evaluation period system response of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study-II.

mathematical calculations. So, other parameters are not optimal one and gives poor result compared to the proposed method. It is also analytically proved in this paper that the performance of lbest HS algorithm is better than basic HS algorithm. Therefore, it can be observed from Table 1 that, the performance of hybrid LBHS based L_1 adaptive controller is superior to other control strategies considered in this paper in terms of integral absolute error (IAE). Due to optimal parameter setting, the proposed method requires control signal with lower magnitude than the other control strategies to track the desired trajectory. The control signal exhibits some oscillations of small amplitude due to the adaptation of time varying uncertainties and disturbances continuously. In case of error signal, amplitude reduces ten times with some oscillations of very small magnitude in hybrid LBHS controller than conventional L_1 adaptive controller. The Proposed method generates control signal with same frequency that of the uncertainties and disturbances, as shown in Fig. 9. Perfect estimation of uncertainties ensure perfect cancellation of them and assure good performances.

4.1. Simulation case study

The noise rejection capability of the tuned controllers are evaluated with the introduction of 10 dbw Gaussian noise to the system under control. The results show, in Table 1, that the proposed hybrid lbest HS based L_1 adaptive control strategy

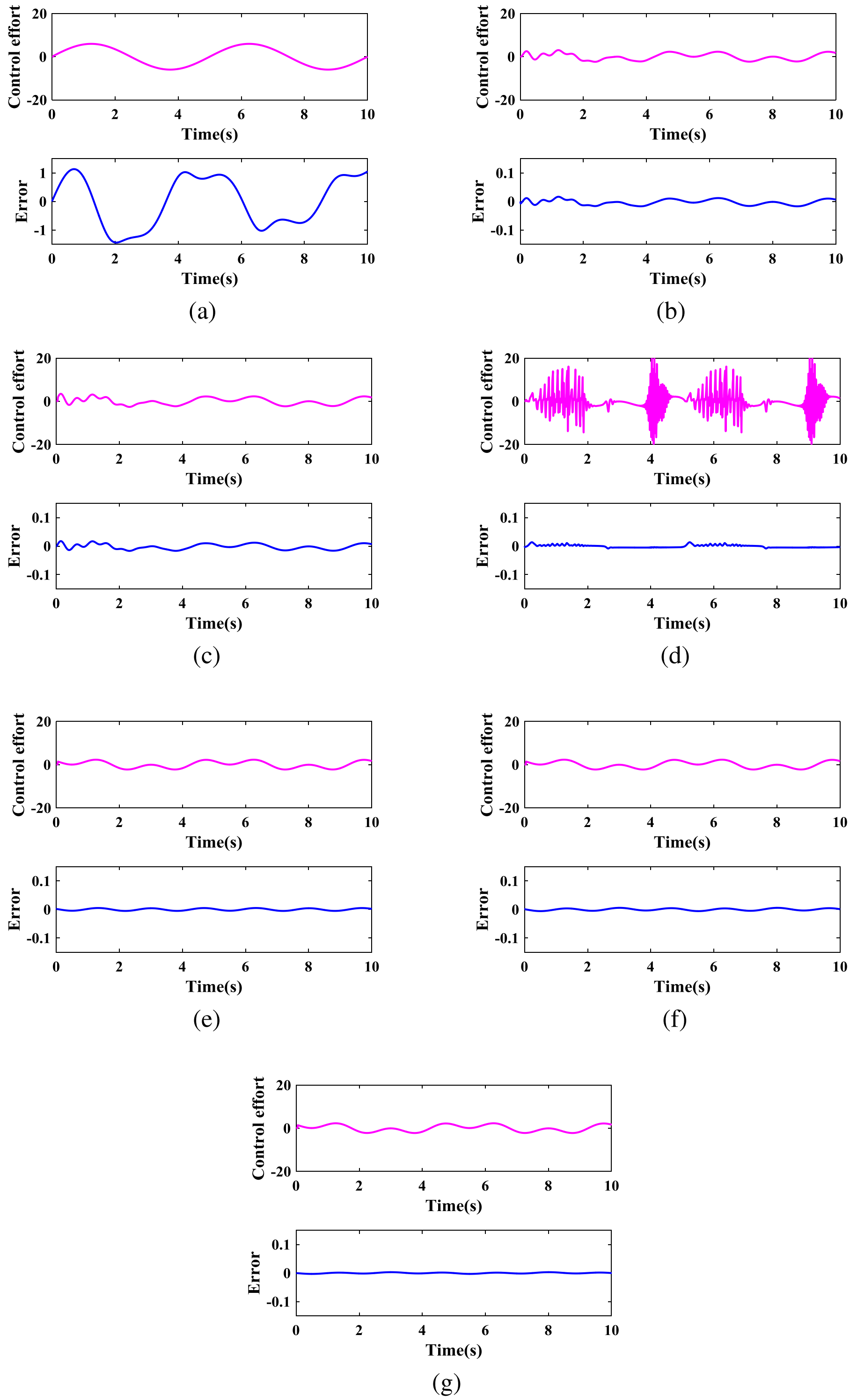

Fig. 12. Evaluation period control effort and error signal of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study-II.

evolves as the best candidate among the other variations studied here. Fig. 10 shows, without further tuning the performances of LBHS based state feedback tracking controller gives poor performance. Performance of L_1 adaptive controller also degrades when noise enters into the system. At the same time, the proposed method gives very good transient as well as steady state performance.

Case study-II

A spring–mass–damper system is examined in this case study and is given as:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.02x_1 - 0.67x_1^3 - 0.1x_2^3 + u(t) \\ y &= x_1 \end{aligned}\right\} \tag{54}$$

Table 2
Comparative study of different control strategies for case study-II.

Control strategy	IAE value			
	For disturbance			For 10 dbw noise
	Best	Average	Std. Dev.	
LBHS based state feedback tracking	7.8562	7.8632	0.0073	14.7072
L_1 adaptive [10,11,21]	0.0813	–	–	0.2694
PID augmented L_1 adaptive [22]	0.0800	–	–	0.2560
Fuzzy feedback filter L_1 adaptive [23]	0.0405	0.1894	0.1255	0.2813
HybdPSO based L_1 adaptive	0.0311	0.4851	0.0175	0.1625
HybdHS based L_1 adaptive	0.0305	0.0328	0.0016	0.0529
Proposed HybdLBHS based L_1 adaptive	0.0140	0.0207	0.0034	0.0937

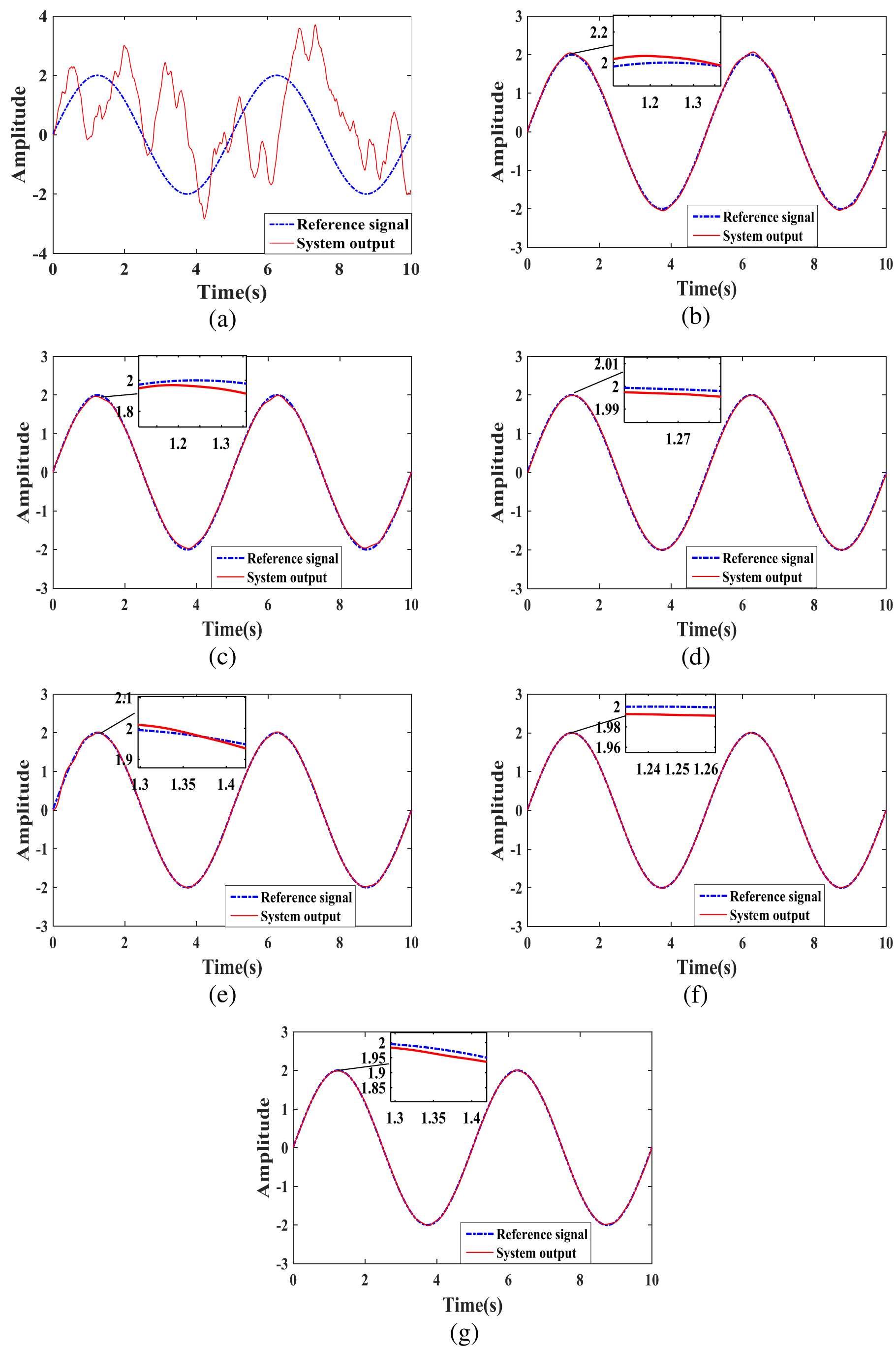

Fig. 13. Evaluation period system response of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study-II with 10 dbw Gaussian noise.

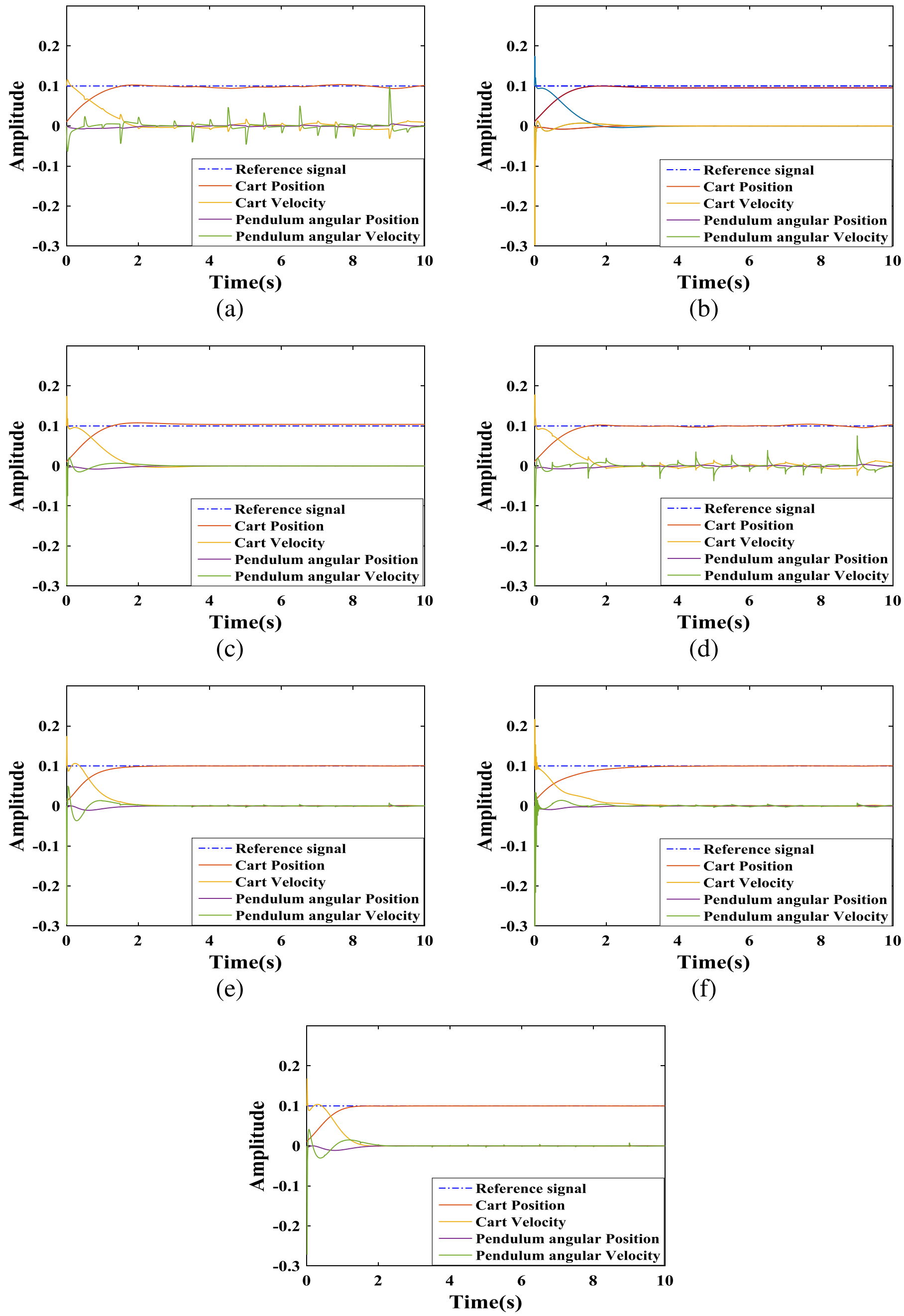

Fig. 14. Evaluation period system response of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study-III.

where, $d(t)$ represents time varying disturbance and noise, similar as case study- I. The control input $u(t)$ is force and the output y is displacement, all are in S.I. unit. The control objective is to track the reference trajectory $y_m = 2 * \sin t$.

This case study also performed as case study – I with all similar paraphernalia. The values of adaptation gains for L_1 adaptive control strategy are evolved as $\underline{\Gamma} = 40{,}000\ 42{,}700\ 39{,}400$ to achieve the tracking performance. A comparative study of different control strategies are presented in Table 2 and Figs. 11 to 13 show the tracking performances for this case study. Table 2 illustrates that, the performance of hybrid *LBHS* controller is better than other control strategies. From Fig. 12, it can be said that the amplitude of control effort required to track desired trajectory

Table 3
Comparative study of different control strategies for case study-III.

Control strategy	IAE value		
	Best	Average	Std. Dev.
LBHS based state feedback tracking	0.0724	0.0942	0.0139
L_1 adaptive [10,11,21]	0.0465	–	–
PID augmented L_1 adaptive [22]	0.0390	–	–
Fuzzy feedback filter L_1 adaptive [23]	0.0217	0.0233	0.0017
HybdPSO based L_1 adaptive	0.0019	0.0024	0.0005
HybdHS based L_1 adaptive	0.0018	0.0064	0.0058
Proposed HybdLBHS based L_1 adaptive	0.0009	0.0033	0.0020

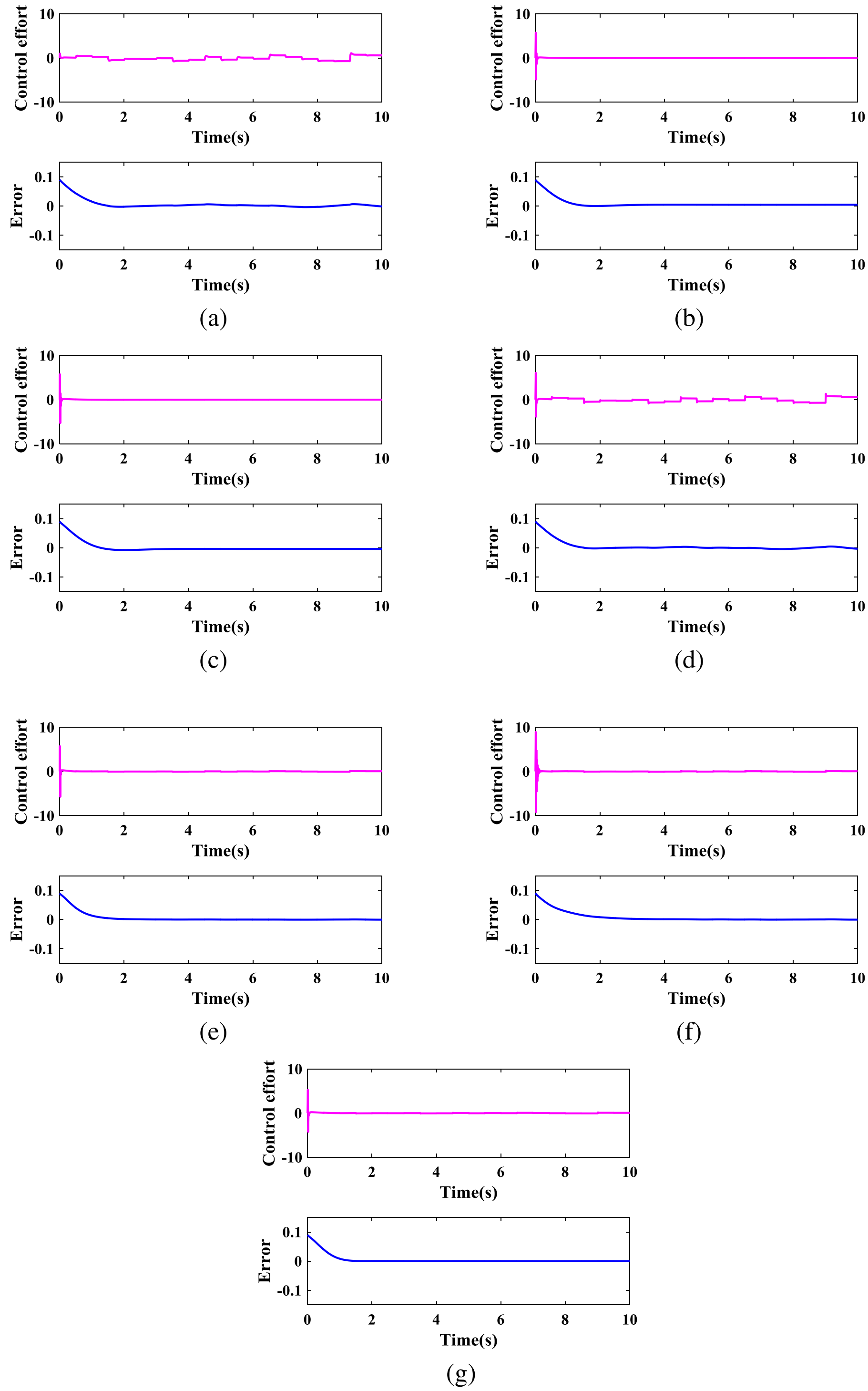

Fig. 15. Evaluation period control effort and error signal of (a) LBHS based state feedback tracking, (b) L_1 adaptive, (c) PID augmented L_1 adaptive, (d) Fuzzy feedback filter L_1 Adaptive, (e) HybdPSO, (f) HybdHS, (g) HybdLBHS controller for case study-III.

is less in the proposed method and it also gives quick transient performance. To adapt the uncertainties, other methods take more time than the proposed method and hence give random control signal at the transient period. In Fig. 12, it is also shown that steady state error is also minimized in our proposed method. The performance of the control schemes with 10 dbw Gaussian noise are shown in Figs. 13 and 13(a) shows that, the noise rejection characteristics of LBHS based state feedback tracking controller is extremely poor. Further, from Table 2 and Fig. 13(g), it can be stated that in terms of noise rejection phenomenon also the proposed HybdLBHS control scheme outperforms all other contemporary control schemes compared in this paper.

Case study-III

In this case study, the proposed method also demonstrates its applicability on a multi input multi output (MIMO) system. The required framework to control a MIMO system is elaborated at first. Let, the MIMO dynamics is given as:

$$\dot{\underline{x}}(t) = A_m\underline{x}(t) + b(\underline{\omega}\,\underline{u}(t) + \underline{\theta}^T(t)\underline{x}(t) + \underline{\sigma}(t)),\ \underline{x}(0) = \underline{x}_0 \tag{55}$$

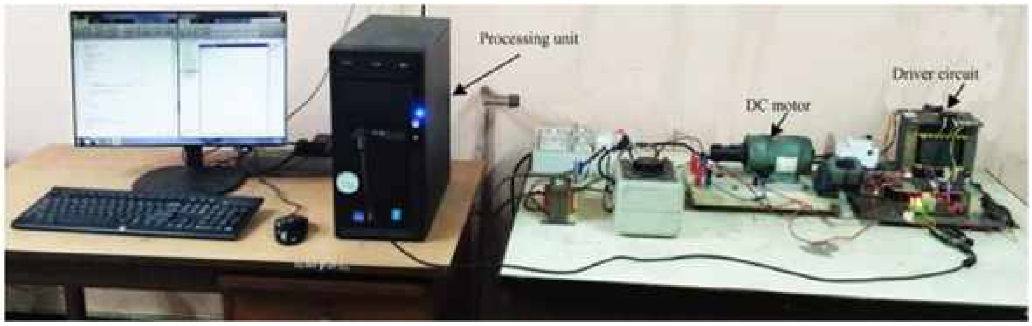

Fig. 16. Experimental arrangement for DC motor speed control.

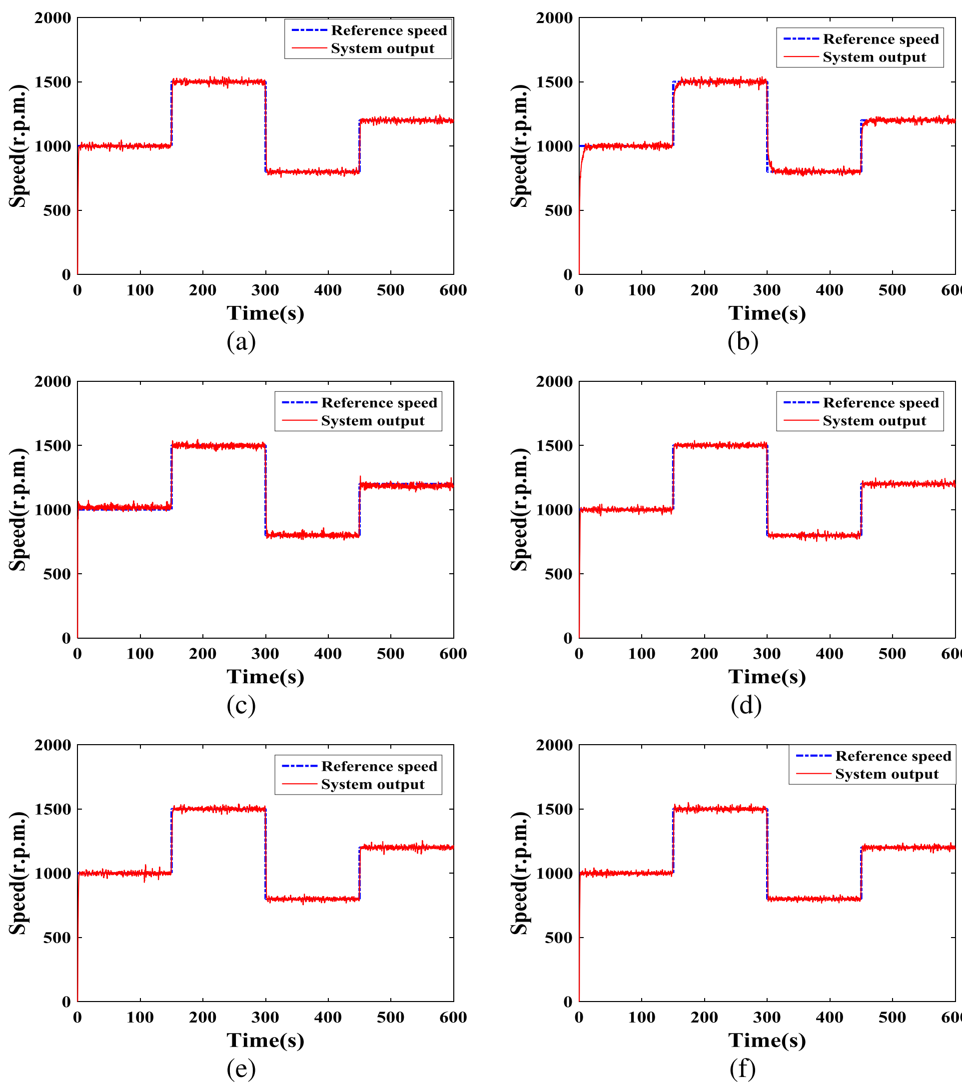

Fig. 17. Evaluation period system response of (a) L_1 adaptive, (b) PID augmented L_1 adaptive, (c) Fuzzy feedback filter L_1 Adaptive, (d) HybdPSO, (e) HybdHS, (f) HybdLBHS controller for case study-IV with input-I.

$$\underline{y}(t) = c^T \underline{x}(t) \tag{56}$$

where, $\underline{x}(t) \in \Re^n$ is the state vector, $\underline{u}(t) \in \Re^m$ and $\underline{y}(t) \in \Re^k$ are the input and output of the system, $A_m \in \Re^{n\times n}$ is system matrix, $b \in \Re^{n\times m}$ and $c \in \Re^{k\times n}$ are known constant vectors. $\underline{\omega} \in \Re^{m\times m}$, $\underline{\theta}(t) \in \Re^{n\times m}$ and $\underline{\sigma}(t) \in \Re^m$ are unknown constant, time varying uncertainty and disturbance present in the system.

An MIMO unstable non-minimum phase [38] 4th order inverted pendulum with cart system with time varying disturbances is chosen here. The system dynamics is given as [39]:

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-mg\cos x_3 \sin x_3 + ml \sin x_3 x_4^2 + u}{M + m\sin 2x_3} + d_1(t) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{-ml\cos x_3 \sin x_3 x_4^2 + (M+m)g\sin x_3 - u\cos x_3}{Ml + ml\sin 2x_3} + d_2(t)
\end{aligned}\right\} \tag{57}$$

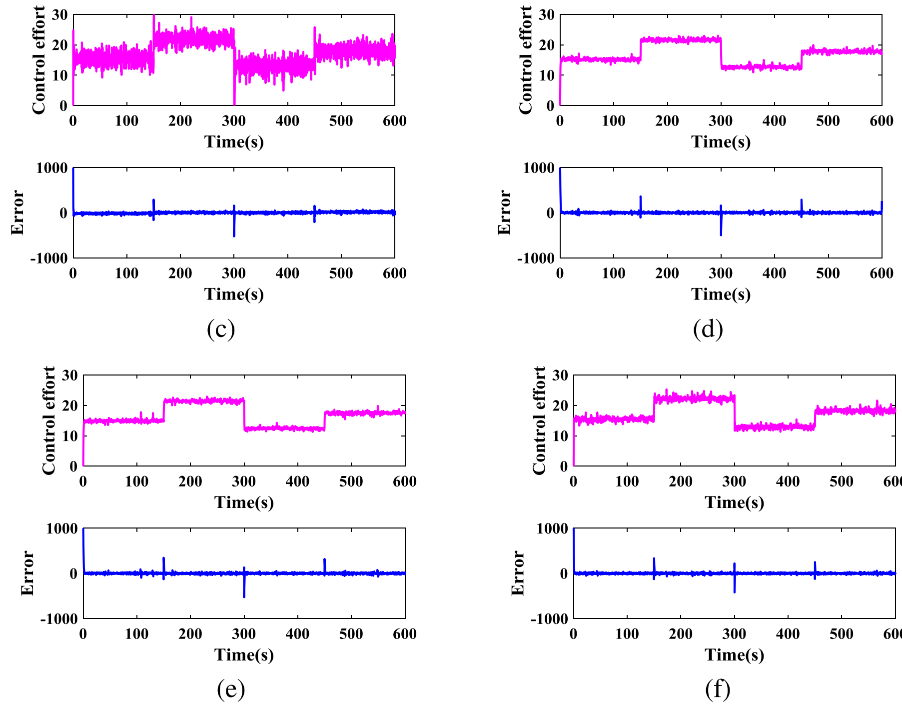

Fig. 18. Evaluation period control effort and error signal of (a) L_1 adaptive, (b) PID augmented L_1 adaptive, (c) Fuzzy feedback filter L_1 Adaptive, (d) HybdPSO, (e) HybdHS, (f) HybdLBHS controller for case study-IV with input-I.

where, x_1, x_2, x_3, x_4 are the cart position, cart velocity, pendulum angle, pendulum angular velocity respectively, u is the required force, all are in S.I. unit. $d_1(t)$ and $d_2(t)$ represent the disturbance with random amplitude varying between $[-1\ 1]$ and $[-4\ 4]$ respectively, with a time period of 0.5 s. In comparison with (1), in this case study the constant gain component of $u(t)$ is taken as $\underline{\omega}$ and the time varying gain component is considered as disturbance $\underline{\sigma}(t)$. Here the aim is to produce a control law that can stabilize the unstable non-minimum phase system to a reference position (here, the reference cart position is 0.1*$u(t)$) and can properly cancels out uncertainties and disturbances. Here the control law of L_1 adaptive controller is chosen as [10,11]:

$$\underline{u}(s) = -kC(s)\left[\hat{\eta}(s) - k_g r(s)\right] - \underline{K}\underline{x}(t) \tag{58}$$

where, $\underline{K} \in \Re^{n\times m}$ is the state-feedback matrix. By putting the value of $\underline{u}$ from (58) in (55), the closed loop system model becomes:

$$\begin{aligned} \dot{\underline{x}}(t) &= A_0\underline{x}(t) + b(-\underline{K}\underline{x}(t) + \underline{\omega}\ \underline{u}(t) + \underline{\theta}^T(t)\underline{x}(t) + \underline{\sigma}(t)) \\ &= (A_0 - b\underline{K})\underline{x}(t) + b(\underline{\omega}\ \underline{u}(t) + \underline{\theta}^T(t)\underline{x}(t) + \underline{\sigma}(t)) \\ &= A_m\underline{x}(t) + b(\underline{\omega}\ \underline{u}(t) + \underline{\theta}^T(t)\underline{x}(t) + \underline{\sigma}(t)) \end{aligned} \tag{59}$$

The value of $\underline{K}$ has to be chosen properly so that, $A_m = (A - b\underline{K})$ will become stable Hurwitz matrix.

Here, the mass of pivot $M = 1$ kg, mass of pendulum rod $m = 0.1$ kg, length of the pendulum rod $l = 0.3$ m is considered [39]. The mass of pivot is a finite large ($M \gg m$) unknown value within a specified bound so that u will belong within the actuator saturation limit. In this work, for above specification, the control signal $u \in [-20, 20]$ which is within the actuator saturation limit [39].

Different control strategies, same as the above two case studies, are done to show the tracking performance of cart position and the results are given in Table 3. In case of unstable, non-minimum phase system, all of its states have to be stabilized. Therefore, the nature of all the four states for different control strategies are shown in Fig. 14. Control effort required and error signal are given in Fig. 15.

4.2. Experimental case study

Case study-IV

A DC motor with loading arrangement is chosen to demonstrate the usefulness of the proposed hybrid adaptive control strategies. The state model of DC motor can be written as:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\left(\frac{BR_a}{JL_a} + \frac{K_bK_T}{JL_a}\right)x_1 - \left(\frac{R_a}{L_a} + \frac{B}{J}\right)x_2 \\ &\qquad + \frac{KK_T}{JL_a}u(t) \\ y &= x_1 \end{aligned}\right\} \tag{60}$$

where, $[x_1x_2] = [\omega\dot{\omega}]$ and the output y is the angular speed of the motor (in rad/s) i.e. ω.

The unknown motor parameters are: armature resistance (R_a), armature inductance (L_a), inertia constant (J), damping factor (B), motor back emf constant (K_b), motor torque constant (K_T) and motor driver circuit constant (K).

In contrast with the control strategies discussed so far for benchmark processes in simulation, where the system parameters were known, this situation provides a real-time challenge for the proposed hybrid concurrent model of L_1 adaptive controller, where no *a priori* knowledge is available regarding the system under control. The parameters of the DC motor are therefore, at first, estimated with the help of HS algorithm. With this estimated parameters the DC motor has got uncertainty in the model, natural disturbances as well and that is used in real-time operation. To perform the parameter estimation, the open-loop test data of input voltage and its corresponding speed of the DC motor are acquired for ten minutes. The speed measurement is carried out using a tacho-generator mounted on the motor shaft. Here the HS algorithm determines the best harmony vector containing the unknown parameters for which difference between the model output and the experimental output data, for the same input, accumulated over the entire set of input–output experimental

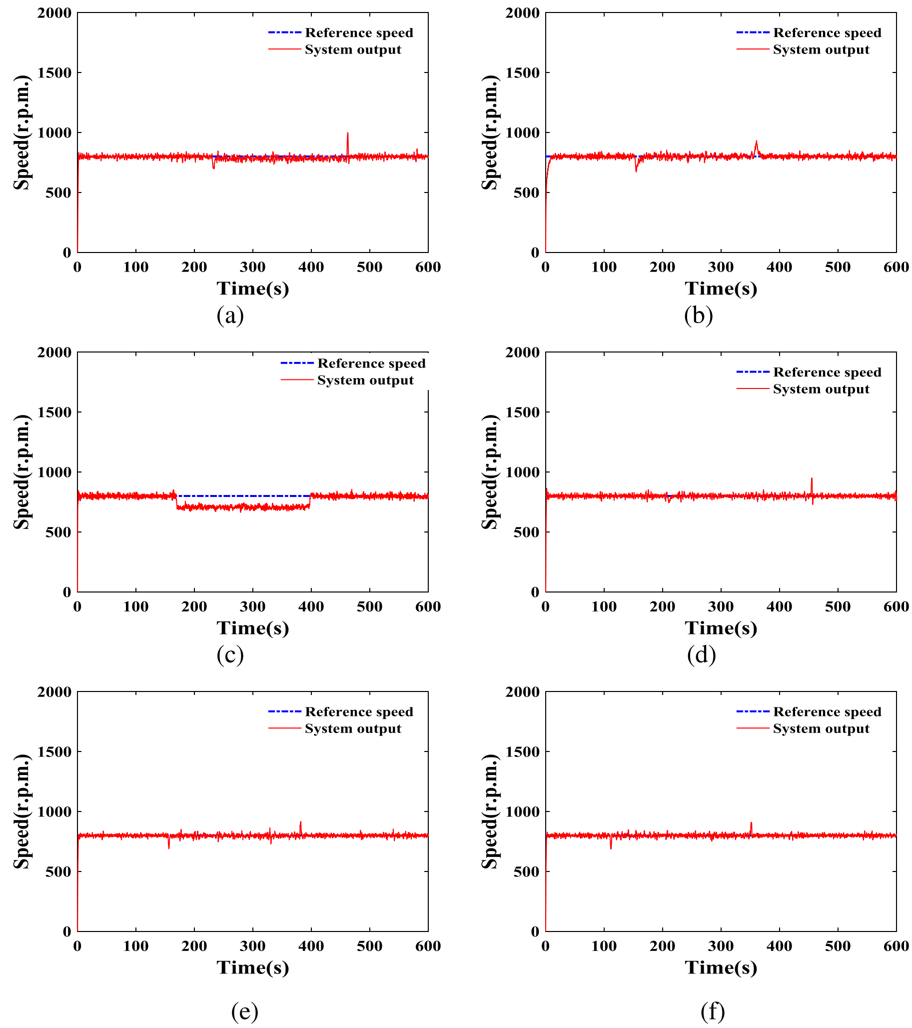

Fig. 19. Evaluation period system response of (a) L_1 adaptive, (b) PID augmented L_1 adaptive, (c) Fuzzy feedback filter L_1 Adaptive, (d) HybdPSO, (e) HybdHS, (f) HybdLBHS controller for case study-IV with input-II.

data, is found minimum. Five test cases are carried out to obtain the results finally.

Here, the speed control of a 25 W, 50 V, 3000 rpm DC motor is tested where the control signal is generated in MATLAB®. That controlled voltage input is then fed back to the DC motor through a driver circuit and speed of the motor is taken as the output through a tacho-generator. This speed is converted into voltage with conversion ratio 2 V/1000 rpm and is fed back to the MATLAB® to produce required control action. The motor is equipped with an aluminium disc mounted on the shaft for eddy current loading, as shown in Fig. 16. The LBHS and PSO [37] algorithms are separately combined with the L_1 adaptive controller, to develop the proposed hybrid concurrent controllers in such a fashion that, it can achieve the automation in the design process and also guarantee the stability of the designed controllers.

For this experimental study, two different reference trajectories are considered, as follows:

Input reference-I: A four step variable DC signal given as:

$$r(t)=\begin{cases}1000u(t)\ 0\le t\le 150\ \text{s.}\\ 1500u(t)\ 150<t\le 300\ \text{s.}\\ 800u(t)\ 300<t\le 450\ \text{s.}\\ 1200u(t)\ 450<t\le 600\ \text{s.}\end{cases}$$

Input reference-II: A single step fixed DC signal with 100% loading, given as:

$$r(t)=800u(t)\ 0\le t\le 600\ \text{s.}$$

The unknown constant, time varying uncertainties and disturbances are predicted and adapted offline with input reference-I. Then the tuned controller is utilized to evaluate the system dynamics online with both the input references. The values of adaptation gains for L_1 adaptive control strategy are as $\underline{\Gamma}$ = 21,000 24,700 20,100 to achieve the tracking performance.

A comparative study of different control strategies with different reference input for this case study is presented in Table 4 and Figs. 17 to 18 show the tracking performances for input-I. Similarly, Figs. 19 to 20 show the tracking performances for the input-II. From Table 4 it is evident that the hybrid concurrent model of lbest HS algorithm outperforms the other controller design strategies considered in this paper. Also, comparing the responses of Fig. 19 it is clear that during loading the tracking performance in case of Hybrid LBHS based L_1 Adaptive is much better than conventional L_1 Adaptive controller. Therefore, from results it is evident that, at the time of online evaluation, if the time varying uncertainties and disturbances present in the system belong within the actuator saturation limit, then the tuned lbest

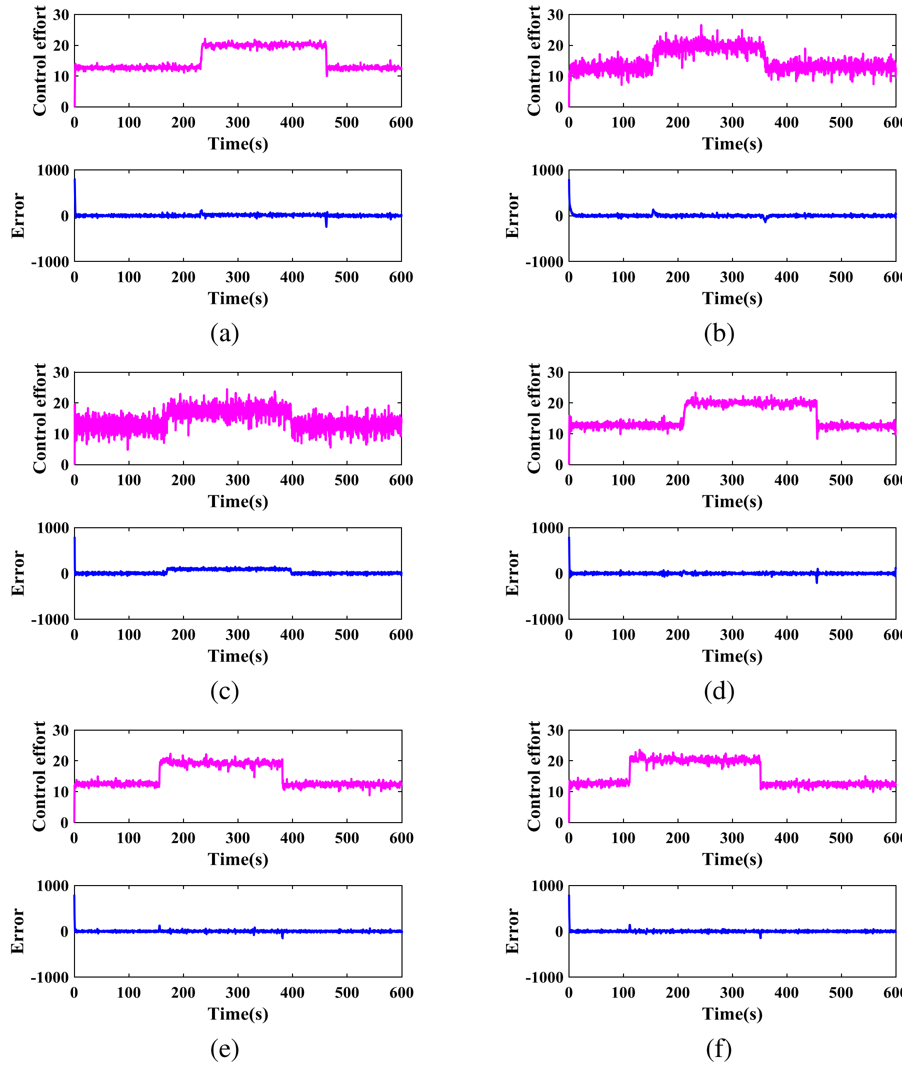

Fig. 20. Evaluation period control effort and error signal of (a) L_1 adaptive, (b) PID augmented L_1 adaptive, (c) Fuzzy feedback filter L_1 Adaptive, (d) HybdPSO, (e) HybdHS, (f) HybdLBHS controller for case study-IV with input-II.

Table 4
Comparative study of different control strategies for case study-IV.

Control strategy	IAE value	
	Input-I	Input-II
L_1 adaptive [10,11,21]	10 002.30	6038.56
PID augmented L_1 adaptive [22]	11 291.02	3786.42
Fuzzy feedback filter L_1 adaptive [23]	13 554.84	22 236.07
HybdPSO based L_1 adaptive	9989.54	1766.42
HybdHS based L_1 adaptive	9994.71	1838.63
Proposed HybdLBHS based L_1 adaptive	9328.89	1631.86

HS based L_1 adaptive controller is capable of providing stable tracking performances.

5. Conclusion

The present work proposes a novel scheme for controlling a class of non-linear systems with uncertainty and external disturbances utilizing the stable optimal L_1 adaptive robust tracking controllers, designed by hybridizing L_1 adaptive control scheme and a lbest variant of HS algorithm based metaheuristic optimization. The stability of the proposed controller is guaranteed by the Lyapunov theory and the required automation is accomplished by employing lbest HS algorithm based optimization technique. The robustness issue is being taken care of by selecting the filter parameters properly and the good transient performance with fast adaptation is obtained by selecting optimal adaptation gains. The explorative and exploitative behaviour of the proposed lbest HS based hybrid L_1 adaptive controller is precisely detailed and convergence of the overall process has been established analytically. The proposed strategy is successfully implemented for both in simulation and in real-time experimentations. It is successfully validated that, the proposed lbest HS based hybrid concurrent L_1 adaptive control scheme evolved as a superior approach compared to other competing controllers, designed using basic HS- and PSO-based approaches.

A prospective research initiative may focus on higher order filter design for L_1 adaptive controller to mitigate the high frequency contamination in control signal due to high adaptive gains. The effectiveness of the proposed lbest HS based L_1 adaptive controller architecture for higher order non-linear systems, time delay systems, systems with Markovian jump, interconnected MIMO systems may be studied for simultaneous adaptive and robust control option. Moreover, the proposed control scheme may be further extended to cope with the dynamics of mathematically ill-defined systems.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Proof of Lemma 2.

Let, M be the iterative matrix of Jordon form $M = ZJZ^{-1}$. where $J = diag(J_0, J_1)$.

Then for nth iteration it become $M^n = ZJ^nZ^{-1}$.

Now the spectral radius of iterative matrix is $\rho(M) = \rho(J)$. If $\rho(M) = \rho(J) < 1$ then J^n converges to 0 [40] and as well as M^n converges to 0 (i.e. $\lim_{n\to\infty} \|M^n\| = 0$).

Now, if $\rho(M) = \rho(J) > 1$, then $(J^n)_{ii} = \lambda^n$, where λ is an Eigen value with $|\lambda| > 1$. Now, if ξ is the ith column and ξ' is the ith column of Z^{-1}, then

$$\xi' M^n \xi = \xi' Z J^n Z^{-1} \xi = \lambda^n$$

therefore, $\lim_{n\to\infty} \left|\xi' M^n \xi\right| = \infty$ and $\lim_{n\to\infty} \|M^n\| = \infty$.

The proof is completed. ■

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