Process Optimization of Digital Conjugate Surfaces: A Review

Pagidi Madhukar, Guru Punugupati, N. Selvaraj and C. S. P. Rao

Abstract Complex surface methods are widely used in various industrial tools such as cutting tools, shell of television, turbine blades etc. Researchers tend to concentrate on conjugate surfaces, which can be produced by digital method to optimize any process. Conjugate surface deals with relative motion. Most of the cases pertain to power transmitting devices such as gears. Providing conjugate surface is very difficult, and to overcome this problem, digital conjugate method has been introduced. It produces event complex conjugate surfaces. In such cases, researchers have focused on Digital Gear Tooth Surface (DGTS). It causes conjugate motion between the gear teeth, which is represented through discrete points, and with these discrete points, the digital surface of conjugate moments is resolved. Computer simulated examples generate and machine the non-standard and complex shapes of digital conjugate tooth surfaces. This technique is not only useful for machining discrete digital gear tooth surfaces and gear tooth surface with complex design, but also other 3D digital surfaces with optimized process.

Keywords Conjugate surface · Digital gear tooth surface · Complex surface

1 Introduction

Technology continues to improve every day according to the demand for modern precision machinery. In such machineries, gear components play a crucial role as power transmitting mechanism. Therefore, to meet the demand and manufacturing of these components requires complicated analysis and treatment of surface engagement. Computer technology helps to develop discrete treatment method.

General purpose CAD/CAM system are relatively mature in modeling various complex surface shapes but they are quite weak in further treatment of complex

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surface models. Nowadays, direct machining is being generated using algorithms via two techniques. One of them is Area by Area Machining [1, 2]. According to this technique, typical complex surface assigns are separated from not so complex components and a large or small tool is selected respectively. The second technique involves intersection of planes on the surface to reach the above mentioned objective. For machining of parallel plane strategy, *Z*-map model is used [3]. The intersect of mean least square surface (MLS) is also used to drive planes [4].

Modern engineering technology is encouraging machine-building aligning towards synthesis, digitization and intelligence. This accelerates the research on generating method of digitized surface. The generating digitized surface problem solving depends on the digitized conjugate surface theory [5]. Theory of conjugate surface is a novel technology which examines mutual conversions and the relationships between paired motions and paired geometric graphs under conditions of mechanical transmission and machinery processing. It involves mechanics, mechanism, differential geometry, and can be used for the design of cutting tool contour, cam contour as well as gear tooth face. It can also be used for the synthesis of mechanism, motion analyses, processing simulation and so on.

1.1 Conjugate Surface Concept

To study the theory of conjugate surface, both analytic and digitized surfaces are important and conjugate relationship as well as conditions play a critical role (Fig. 1).

$\Sigma 1$ and $\Sigma 2$ are two arbitrary conjugate surfaces, If $\Sigma 1$ is considered as mother surface,

$S1$ (Ol $X1$ Yl Zl) and $S2$ ($O2$ $X5$ $Y2$ $Z2$) are the two coordinates.

Surfaces 1 and 2 are associated at point M where $M1 = $ M2

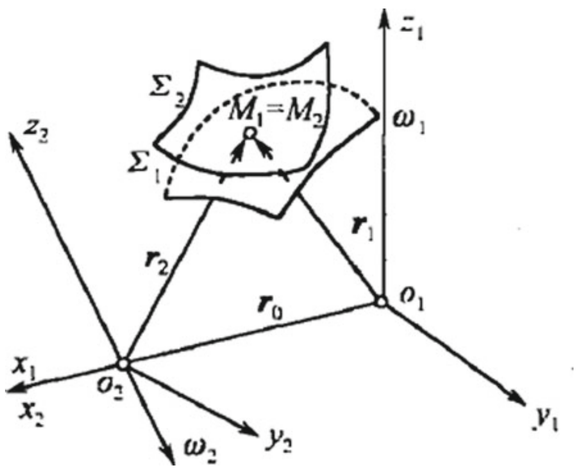

Fig. 1 Two surfaces of conjugate body motion

Position vectors are $r1$ and $r2$
Normal vectors are $n1$ and, $n2$

Surface $S1$ is connected with $\Sigma 1$, and surface $S2$ with $\Sigma 2$. Similarly, position vectors $r1$ point to $\Sigma 1$ and $r2$ point to $\Sigma 2$. Unit normal vectors $n1$ have point rl on surface $\Sigma 1$ and $n1$ point $r2$ on surface $\Sigma 2$ and $\Sigma 1$ considered as mother surface, the equation may expressed with u & v; then $r1 = r1(u, v)$

Surface $\Sigma 1$ changes as per law, ɵ is dependent parameters, and it can form a series of surfaces (surface cluster). The equation of these surfaces as followed:

$$r1 = r1(u\ v\ \Theta)$$

u & v are the mother surface geometrical parameters, and ɵ is the motion parameter. Figure 2 shows the conjugate profiles and Fig. 3 shown the various applications of conjugate surfaces.

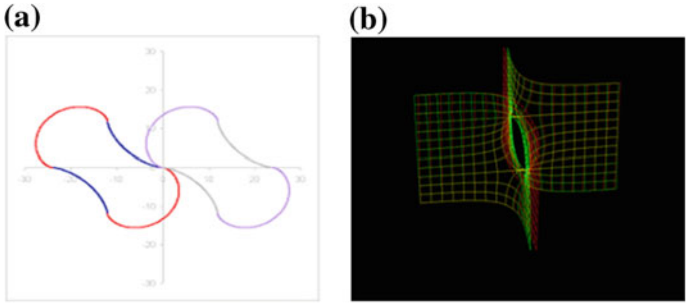

Fig. 2 **a** Conjugate motion of a gear profile, **b** Conjugate profile at contact surfaces

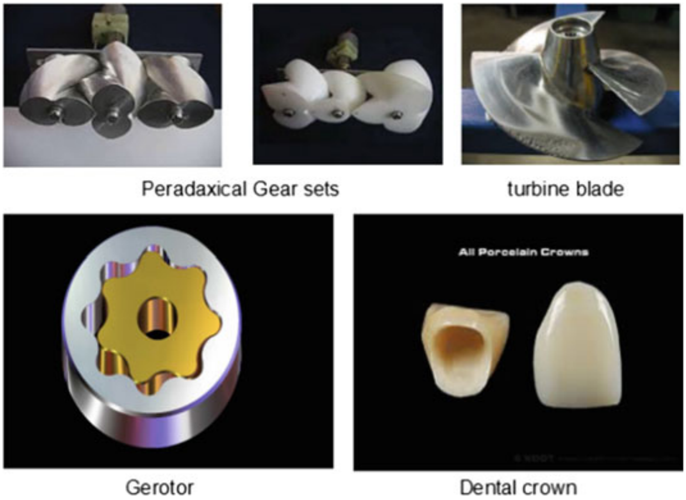

Fig. 3 Various applications of conjugate surfaces

Various conjugate surfaces applications such as Paradaxical gear set, turbine blade, gerotor, dental crowne figures are shown in below.

Researchers have tended to focus on various tooth profiles for improvement of performance over the last decade. Author Litvin et al. [6] investigated and proposed a parabolic tooth profile to minimize transmission errors. Tsay et al. [7, 8] suggested a changed form for helical gears with circular as well as involute tooth. Zhang [9] examined double involute profiles with a transition among them. The scope possibly extended to multiple involute profiles with various pressure angles. Ariga et al. [10] represented a tooth profile with a combination of circular arc and involute. And these combinations are closer to the idea of enforcing splines. Komori [11] developed and proposed a design for a spur gear with zero relative curvature at lots of contact points.

To avoid manufacturing difficulties in conjugate surfaces, digital surface approach was introduced and is widely used. A digital surface implies that, the digitalized points of a adjacent surface can be saved as a three dimensional matrix on a computer. Digital surface is also defined as a point set of Σm which appears like a surface rather than a curve or solid based object. This method describes surfaces in terms of discrete points. Digital surface method not only meets the requirement of complex shape representation as well as geometric design, it also helps in product data exchange and leads to information transmission. Therefore it's widely used in manufacturing and mechanical designs. There are two techniques to find digital surfaces. "Modern Design" is one technique, which includes boundary element method and finite element method. The second one is 3D-digital measurement of manufactured part or real objective models.

2 Literature Review

Luo [12] creatively designed and represented high load capacity cosine gear tooth profiles. It describe the cosine gear generation principle and mathematical model which consist of conjugate tooth profile. Cosine tooth profile and line of action equations are well established based on meshing theory. A new solid model was generated and characterized by computerized simulation such as contact ratio and sliding ratio, bending and contact stresses for this new solid drive model. It was found that the cosine gear showing better performance than the involute gear. The FEM model and stress distribution for each profile is shown in Fig. 4. Ming [13] discussed digital conjugate surfaces for solving computationally intensive problems by dimension reduction interpolation method and also alternative surface interpolation using the interpolation curve to achieve a conjugate surface normal vector as well as relative velocity of the system. According to Lijun [14], conjugate surfaces theoretical analysis is based on the principle of rotation, its beginning being an envelope of derived surface, forming a cylindrical surface of the rotating conjugate surface equation and contact line equation. It explained how the line of contact is drawn by computer and how the round steel leveler rolls and round conjugate motion may be achieved by computer simulation. Gang and Cuo [15] proposed modern computing technology

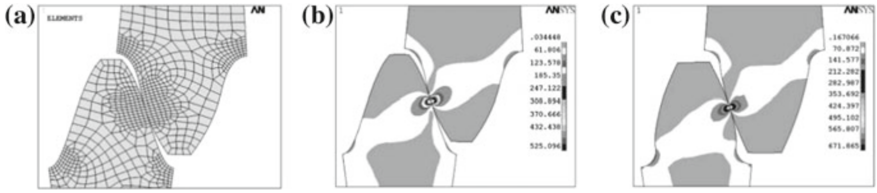

Fig. 4 **a** FEM Model **b** Stress distribution of cosine &**c** Involute profile [12]

and mathematical programming based on the principle of conjugating surfaces. They provided a direct description of the conjugate process with mathematical model and this model is used for both theoretical and digital research. A result obtained by digital simulation method covers the traditional conjugate surface theory of the basic content, adapted to solve interference and non-deformation problems under complex conditions.

Xin et al. [16], proposed a new method of numerical simulation to figure out special problems of conjugate surface in the form of discrete numerical data. The conjugating surface problems are changed into mathematical programs under minimum conditions. In this paper Guo et al. [17] proposed adaptive synthesis to analyze and discretize the conjugate surfaces to design the computer software by taking the conjugation process into account. Ming [18] used MATLAB to solve digitized conjugating surface problems. And he showed that surface interpolation was substituted by curve interpolation, to get the vectors and relative rate of digitized conjugating surfaces. By lower dimension interpolation method, the mathematical model of digitized conjugating surfaces problems were solved and a solution program was developed in MATLAB while the interface was achieved by Visual C++.

Xiao et al. [19], presents a design method for digitized conjugating surface by using MATLAB. The researchers proposed a mathematical model for conjugating surface and solving it according to the conjugating principle and computing feature of MATLAB. They invented a method to extract 1 dimensional arrays from a 3 dimensional array, and its results were optimized by a simple MATLAB function called the design process. Ming [20] has shown how the conjugating surfaces are solved by a mathematical model based on the digital dimension reduction interpolation method. The author used computing software MATLAB for powerful numerical and graphical display capabilities to develop the digital conjugate surface algorithm, with visual C++ to achieve a human-machine interface and processing of data. Masood et al. [21] introduced Direct machining process by providing efficient paths of tool from cloud data, which is saved in STL file format. The primary aim is to attain maximum efficiency in the machining of sculptured surface geometries, accepting complex machining fields. An algorithm was developed to generate various tool paths (Fig. 5a) for direct manufacturing of machining surfaces as well as studied a case study for cloud data (Fig. 5b).

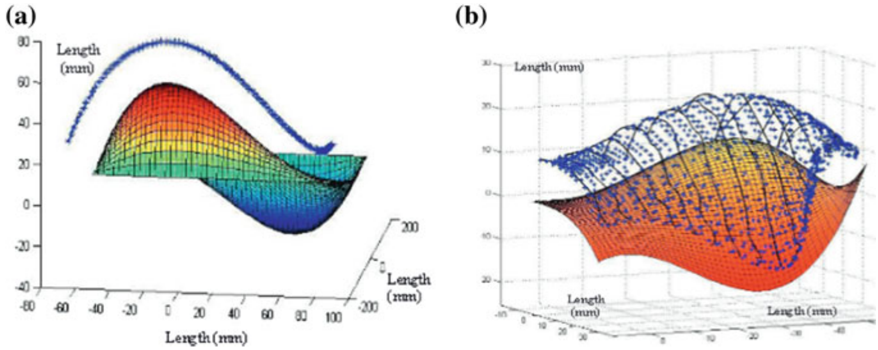

Fig. 5 **a** Tool path **b** Case Study for cloud data on conjugate surface [21]

Wang et al. [22] used machining method for complex surface face gear with hob cutter and CNC machine tools, based on gear hob principle and surface characteristics. Five axis CNC was used for machining complex components. A CNC hob technique process was suggested about the surface orthogonal gear. Yu and Ting [23] presented the first theory as well as exercise of free-form conjugation modeling. They proposed a novel technique of master-slave that extends the conventional computer aided single to dual geometry modeling. They later applied gear geometry to demonstrate the first free-form conjugation modeling method. Litvin et al. [24] developed a simplified approach to determine normal and principal curvatures for complex surfaces where the representation is in three-parameter form. Da Zhun and Liu [25] proposed average comprehensive radius of curvature (ACRC) and a new quality index for characterize the leakage path of the geometrical shape.

Radzevich [26] rectified the problems when machining sculptured surfaces on the multi-axis NC machine. The author developed six requirements and proper considerations for sculptured surface machining (SSM). Chen [27] proposed a pair of line conjugation ruled surfaces which produce instantaneous conjugate movement to one degree of freedom for various succinctness. The base conjugate complex surface is specified to be a minimal surface, a surface with only one parameter for specification of surface curvatures. Wang et al. [28] introduced a generating technique for digital surfaces of gear tooth, and focused on relative conjugate motion between digital gear tooth surfaces and discrete points and also solved non-standard shapes of digital gear tooth surface profiles shapes. As per “pre-calibration error compensation” approach, the machining on final sculpture surface errors are reduced subsequent operations as well as rebuilt the error compensation scenario. These error compensations are represented in Fig. 6 with respect to initial machined results.

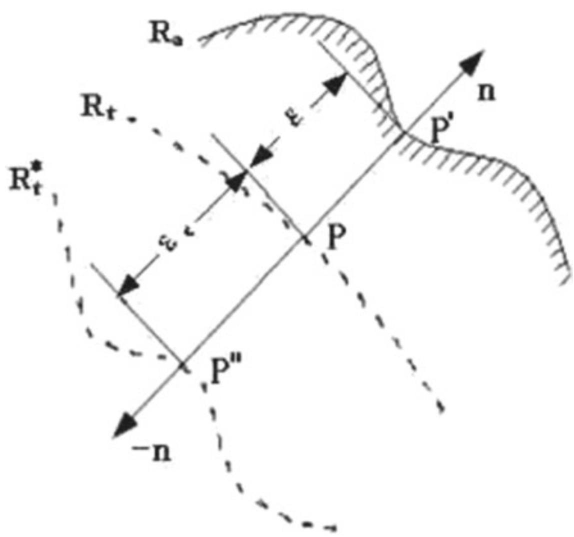

Fig. 6 Error compensations with respect to first machining results [28]

3 Conclusion

The researchers have been trying to solve conjugate surface using digitized surface method, for which one must construct analysis equation for digitized surface. To achieve this, reduced dimension and decomposition techniques and optimization algorithms can highly decelerate the complication of simplifying conjugate surface. Invariant approach, and rigid-body motions are represented by the curvatures of the surface and rotation in terms of a single surface curvature tensor. All the surface quantities are represented by invariants (tensors as well as vectors) and the outcomes of point and line contacts are directly received based on analysis and tensor algebra. This approach will be helpful for theoretical growth in gearing system theory without reference to coordinate systems; the components can be expressed in the form of a matrix, carried out by clearer and easier processing calculations. With this, other complex conjugate shapes can be easily designed, fabricated and simulated with optimized processes.

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