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Evaluating the Performances of Statistical and Neural Network Based Control Charts

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Abstract. Control chart is used widely in many fields and traditional control chart is no longer adequate in detecting a sudden change in a particular process. So, run rules which are built in into Shewhart $\bar{X}$ control chart while Exponential Weighted Moving Average control chart (EWMA), Cumulative Sum control chart (CUSUM) and neural network based control chart are introduced to overcome the limitation regarding to the sensitivity of traditional control chart. In this study, the average run length (ARL) and median run length (MRL) in the shifts in the process mean of control charts mentioned will be computed. We will show that interpretations based only on the ARL can be misleading. Thus, MRL is also used to evaluate the performances of the control charts. From this study, neural network based control chart is found to possess a better performance than run rules of Shewhart $\bar{X}$ control chart, EWMA and CUSUM control chart.

Keywords: Control Chart, Average Run Length, Median Run Length.
PACS: 89.75.kd

1.0 INTRODUCTION

1.1 Statistical Control Charts

Traditional Shewhart control chart is less sensitive in detecting small shifts in the process mean because it only takes into account the information about the process contained in the last sample observation and ignore the information given by the entire sequence of points [1]. To remedy the limitation, runs rules are included into Shewhart $\bar{X}$ control chart. The sensitivity of Shewhart $\bar{X}$ control chart in detecting small process shifts has been increased and assignable causes can be detected more quickly [2]. In conjunction with that, five run rules schemes are used in this study. They are schemes of two-of-two, two-of-three, two-of-four, three-of-three, and three-of-four. These runs rules incorporated in Shewhart $\bar{X}$ control chart are used to compare with the performance of neural network based control chart. In addition, exponentially weighted moving average (EWMA) control chart and cumulative sum (CUSUM) control chart are included in our study as well. Therefore, the comparison among a wide range of different control charts can be done.

1.2 Neural Network Based Control Charts

An artificial neural network (ANN) is a type of Artificial Intelligence (AI) that composes a number of interconnected units which consist of input/output (I/O) characteristic available in each unit and implements a local function [3]. ANN is flexible, adaptive and can better handle noise and changes in the patterns [4]. ANN has the ability to process information from the response of the neurons and connections to the external output. Neural network model can actually be trained with a set of identified responses so that it could "think in the correct way". Along with the training, every synaptic weight is adjusted until the output could be almost matched with the identified output. Today, ANN is widely used in the statistical field including statistical control charts. ANN such as multi-layered perceptron (MLP) neural networks are often used as estimation tools in place of the classical statistical methods [5]. ANN using an error backpropagation learning rule is an attempt to emulate the extremely parallel and distributed processing of the brain while being examined for use in statistical process control (SPC) [6]. Also, neural network is able to learn the relationships through the data themselves without assuming probability distributions. As more data become available, the performance of neural networks can be improved through training. The design of

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the neural network is straightforward and uncomplicated. Consequently, neural network is effective in diagnosing the abnormal patterns in the control charts if compared with other statistical based control charts [7].

1.3 Objectives of the Study

Since the use of ARL as a sole measure of the performance of a control chart has been widely criticized, thus the use of percentage points of the run length distribution has been recommended [8, 9]. However, the 50th percentile is more widely used because other percentages are usually considered for finding the false alarm rates in situation where the process is in-control. The objective of this study is to compare the ARL and MRL (run length for the 50th percentile) for the types of control charts mentioned for both in-control data and out-of-control data with various shifts in the process mean. Thus, the efficiency of the control chart is evaluated through its response as the shifts in the process mean increases.

2.0 METHODOLOGY

A simulation study is conducted. The required data in this study is generated by using Statistical Analysis System (SAS) version 9.1. The standard normal distribution $N(0, 1)$, X is generated by using the RANNOR function in SAS as shown below:

$$X = mu+sigma*rannor\ (33333) \tag{1}$$

where $mu = 0$, $sigma = 1$. The same method is used to simulate an out-of-control process with a permanent shift in the process mean while the data is generated from a random normal variable $N(\delta, 1)$ where δ is the process mean shifts. Also, the same sets of data are used to perform the training and testing in the neural network model.

Programs were written in SAS to calculate the ARL and the MRL schemes for the types of statistical control charts mentioned which are the run rules schemes of two-of-two, two-of-three, two-of-four, three-of-three, three-of-four, and optimal values of 0.5 and 1.0 respectively for EWMA and CUSUM control charts. The shifted distances from the process mean considered are $\lambda(\mu)$ = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0. The program for neural network based control chart is written so that the ARL and MRL can also be calculated. The same shifted distances of $\lambda(\mu)$ as the statistical control charts are also used. All the estimated values of ARL and MRL are based on 5000 independent simulations run.

2.1 Runs Rules Schemes

For runs rules schemes construction, there are 5 recommended steps to be used as follows [2]:

Step 1 : Desired scheme based on magnitude of shift that is important for quick detection is chosen.
Step 2 : ARL value for in-control state when process shift at zero is decided.
Step 3 : Based on Steps 1 and 2, value of *pU* (probability of a single point falling in the upper region) or *pL* (probability of a single point falling in the lower region) is determined from the corresponding plot provided in Khoo's paper [2].
Step 4 : Sensitivity analysis is performed and the value of *pU* (or *pL*) whose in-control ARL best matches the desired ARL_0 (in-control ARL) is chosen.
Step 5 : Control limit of runs rules scheme can be determined by using the standard Normal table or simple calculation using programming language based on the value of *pU* (or *pL*) from Step 4.

2.2 Two Hidden Layered Multi-Layered Perceptron (MLP) Architecture

MLP is a feed forward neural network. Also, it is a supervised-learning network and its output value is continuous [10]. MLP contains an input layer, one or more than one hidden layer and an output layer. The first layer of MLP is the input layer which receives input from the environment. The middle is the hidden layer that responds to particular features that may appear in the input pattern [11]. An internal representation is provided by the hidden layer(s) so that the network is able to learn any related mapping. Output layer is the last layer of MLP which contains neurons that communicate the output of the system to the user or external environment. MLP is usually used for detecting, forecasting and classifying and is one of the most common networks [12]. In this study, a four-layered neural network model with 15 x 8 x 8 x 1 is adopted. The MLP network used is shown in Figure 1. The nodes in each layer are neurons while the lines connected within the neurons have synaptic weights.

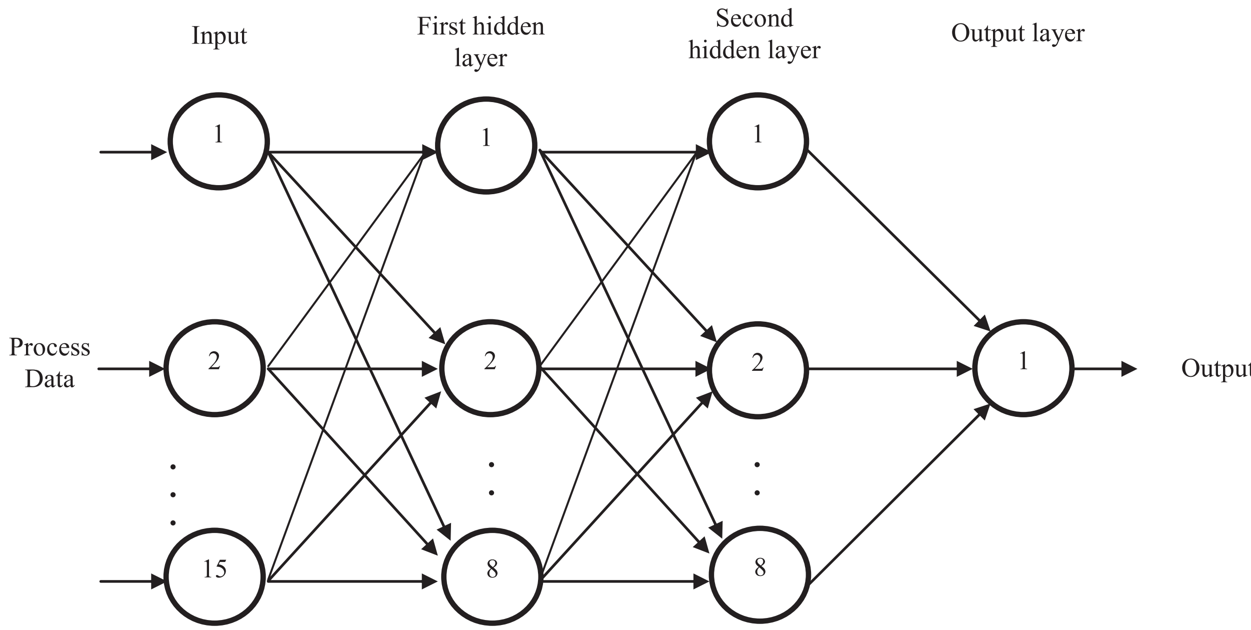

FIGURE 1. MLP with 15 x 8 x 8 x 1 structure

3.0 RESULTS AND DISCUSSION

It is noted that the in-control ARL for statistical and neural network based control chart when $\lambda(\mu) = 0.0$ are fixed approximately at 370. Tables 1 and 2 give the ARL and MRL respectively for Shewhart $\bar{X}$ control chart, five run rules schemes of two-of-two, two-of-three, two-of-four, three-of-three, and three-of-four, EWMA control chart with optimal shift of 0.5 and 1.0, CUSUM control chart for the optimal shift of 0.5 and 1.0 and neural network based control chart. Plots of MRL and ARL for all the control charts as mentioned for various shifts in the process mean, $\lambda(\mu)$ are given from Figure 2 to Figure 5.

TABLE (1). ARL profiles based on in-control ARL=370

Shift in Process Mean	Shewhart	2-of-2	2-of-3	2-of-4	3-of-3	3-of-4	Optimal EWMA at 0.50	Optimal EWMA at 1.00	Optimal CUSUM at 0.50	Optimal CUSUM at 1.00	Neural Network
0.0	374.8808	370.0012	373.3102	370.9230	361.6328	370.6128	370.7686	370.3486	372.9348	367.7218	371.2760
0.5	154.9686	108.6942	100.028	97.4852	91.7636	81.1162	26.4060	30.8976	28.8834	35.4378	21.9028
1.0	43.9564	25.7038	22.9988	22.5102	21.4248	18.2982	10.7678	9.5512	11.4434	9.9364	14.0676
1.5	15.0678	9.1456	8.3060	8.2432	8.4120	7.5316	6.7702	5.4748	7.1418	5.5450	6.2898
2.0	6.3414	4.6026	4.3540	4.3266	4.9336	4.5596	4.9902	3.8606	5.2404	3.8704	3.1812
2.5	3.2220	3.0218	2.9170	2.9460	3.7054	3.5440	3.9844	3.0260	4.1556	2.9944	2.0238
3.0	2.0014	2.3858	2.3594	2.3896	3.2482	3.1968	3.3610	2.5172	3.4816	2.4850	1.3580
3.5	1.4586	2.1404	2.1304	2.1488	3.0622	3.0540	2.9244	2.1914	3.0204	2.1622	1.1442
4.0	1.1922	2.0380	2.0396	2.0504	3.0160	3.0126	2.5642	1.9970	2.6752	1.9570	1.0434

TABLE (2). MRL profiles based on in-control ARL=370

Shift in Proces s Mean	Shewhart	2-of-2	2-of-3	2-of-4	3-of-3	3-of-4	Optimal EWMA at 0.50	Optimal EWMA at 1.00	Optimal CUSUM at 0.50	Optimal CUSUM at 1.00	Neural Network
0.0	263	257	258.5	258	256.5	261.5	256	256.5	257	261	276
0.5	108	74	68	68	65	57	23	24	24	27	13
1.0	31	18	16	16	16	13	10	8	11	9	10
1.5	11	7	6	6	6	6	7	5	7	5	5
2.0	5	4	3	4	4	4	5	4	5	4	2
2.5	2	2	2	3	3	3	4	3	4	3	2
3.0	1	2	2	2	3	3	3	2	3	2	1
3.5	1	2	2	2	3	3	3	2	3	2	1
4.0	1	2	2	2	3	3	3	2	3	2	1

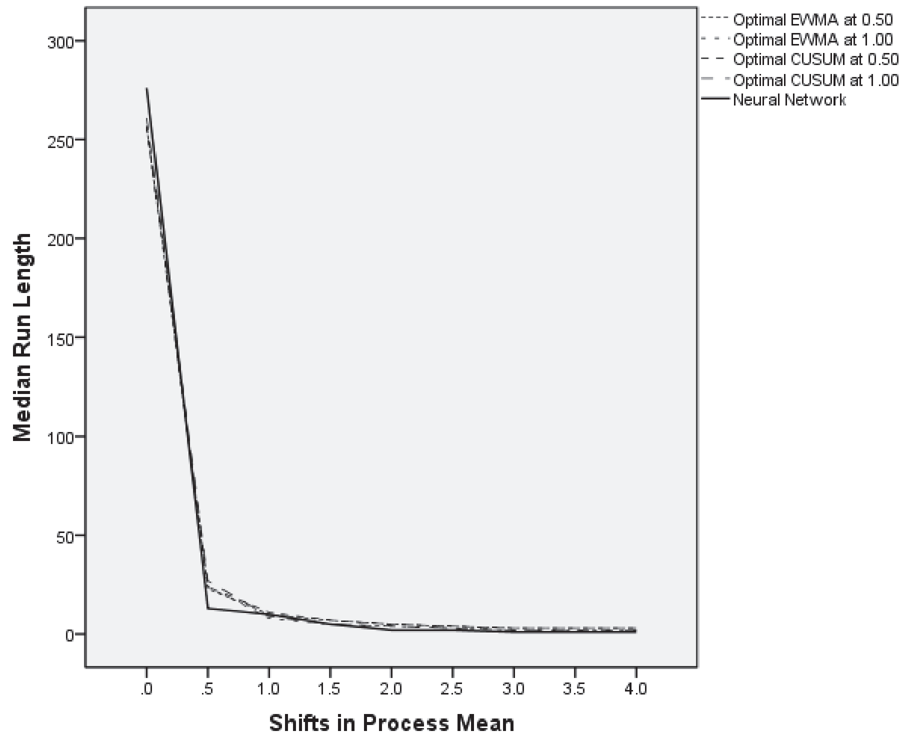

FIGURE 2. Plot of MRL for exponentially weighted moving average (EWMA) control chart with optimal shift of 0.5 and 1.0, cumulative sum (CUSUM) control chart for the optimal shift of 0.5 and 1.0 and neural network based control chart for various magnitude of shifts in λ(μ)

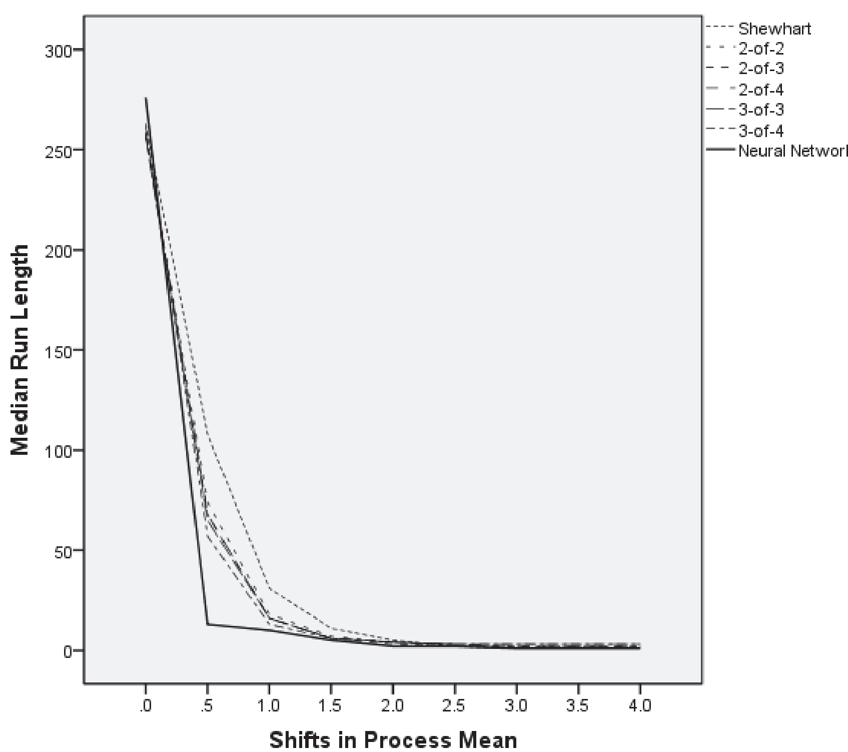

FIGURE 3. Plot of MRL for Shewhart $\bar{X}$ control chart and run rules schemes of two-of-two, two-of-three, two-of-four, three-of-three, three-of-four and neural network based control chart for various magnitude of shifts in λ(μ)

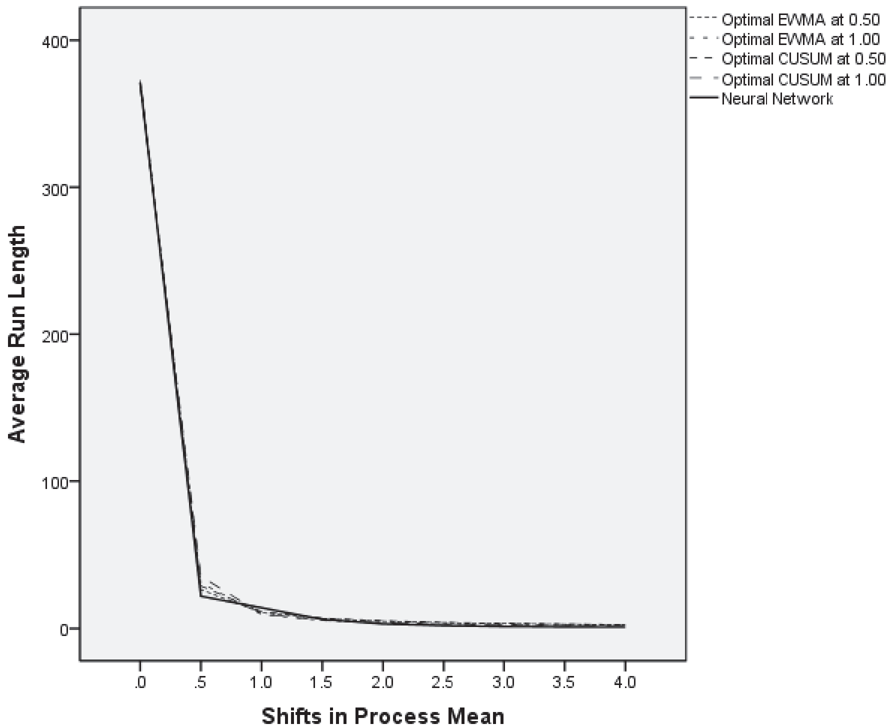

FIGURE 4. Plot of ARL for exponentially weighted moving average (EWMA) control chart with optimal shift of 0.5 and 1.0, cumulative sum (CUSUM) control chart for the optimal shift of 0.5 and 1.0 and neural network based control chart for various magnitude of shifts in λ(μ)

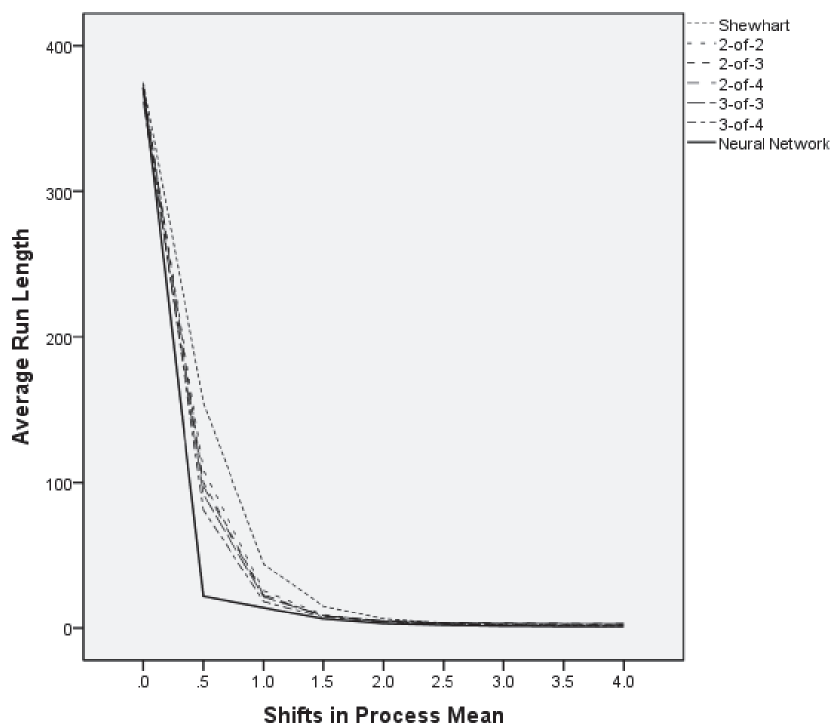

FIGURE 5. Plot of ARL for Shewhart $\bar{X}$ control chart and run rules schemes of two-of-two, two-of-three, two-of-four, three-of-three, three-of-four and neural network based control chart for various magnitude of shifts in λ(μ)

From Tables 1 and 2, it is observed that the in-control MRL for all statistical and neural network based control charts are less than their respective in-control ARL. For example, in the Shewhart $\bar{X}$ control chart, the in-control MRL demonstrates that the run lengths are less than or equal to 263 though the in-control ARL is approximately 370. Therefore, this verifies that the run length distributions are skewed. So, interpretations and conclusion based on ARL alone can be misleading.

Interpretation based on the ARL is further complicated by the fact that the form of the run length distribution changes with the magnitude of the shift in distance, $\lambda(\mu)$ [13]. For example, when the shift in distance, $\lambda(\mu)$ is 2.0, the ARL for Shewhart $\bar{X}$ control chart is 6.3414 whereas half of the run lengths as in the MRL is 5. However, the skewness of the run length distribution decreases as the value of $\lambda(\mu)$ increases. Therefore, interpretations based on the ARL can be confusing since the run length distribution changes with the magnitude of the shift in the process mean, $\lambda(\mu)$.

On the other hand, the MRL would not have the same problem of interpretation. For example, an in-control MRL of 260 would mean that half of all the run lengths are less than 260. Similarly, an-out-of-control MRL of 20 would mean that half of all the run lengths are less than 20. As a result, the MRL is a more reliable quantity for evaluating performance of all the statistical control charts due to the skewness of their in-control run length distributions changes with the magnitude of the shift in $\lambda(\mu)$. Palm [14] has pointed out that the MRL may be more useful than the ARL because run length distributions are highly skewed. Thus, a wide range of comparison for all different types of control charts will be evaluated based on their MRLs so as to obtain the more accurate interpretations.

According to Table 2, it is clearly observed that the neural network based control chart is generally superior in detecting small and large process mean shifts if compared to standard Shewhart control chart, runs rules schemes, EWMA and also CUSUM control chart. This is because neural network based control chart is more sensitive to shifts in process mean. When the shift in process mean is zero which means the process is in-control, it is seen that all different kinds of control charts show a MRL of around 260. However, as the process mean is shifted by 0.5, there are only 13 samples needed by the neural network based control chart for detecting an out-of-control MRL. Moreover, it is relatively higher values of 57, 23 and 24 samples for three-of-four runs rule scheme, EWMA and CUSUM control chart at optimal value of 0.5 respectively. Indeed, these 3 charts are popular in detecting small shifts [1].

Surprisingly, it can be noticed that the neural network based control chart shows a comparable performance with EWMA and CUSUM control charts for shifts of the process mean at 1.0 and 1.5. This is further supported by the plots in Figure 2 that relatively same number of samples are needed by these 3 charts to detect an out-of-control MRL. It is shown that the neural network based control chart possesses an outstanding performance when compared to statistical control charts at the shift in the process mean of 2.0. Furthermore, neural network based control chart is observed to perform better than all other control charts when the process means shift from 2.5 to 4.0, and comparable to standard Shewhart control chart as shown in Table 2.

Though standard Shewhart control chart is well known for large shift detection of a process, but apparently the neural network based control chart is comparable as shown in the out-of-control MRL if compared to standard Shewhart control chart. It is shown by Figure 3 that the plot of standard Shewhart control chart is approximately the same as neural network based control chart when the shifts of process mean is large from 2.5 to 4.0. As a result, it can be said that the neural network based control chart is equally good as standard Shewhart control chart in detecting large shifts in the process mean.

Moreover, the plots shown in Figures 4 and 5 are relatively the same as the plots in Figures 2 and 3 respectively. However, comparisons among all the control charts are measured mainly on Figures 2 and 3 since interpretations based on MRL is more reliable than ARL.

In summary, it is concluded that the neural network approach is more appropriate to be applied in the control charts in order to ensure the quality of a process.

4.0 CONCLUSION

Quality control engineers should possess a good knowledge of different types of control chart. Therefore, it is demonstrated in the earlier discussion the importance of computing MRL of statistical and neural network based control charts as they provide information to evaluate the performances of different types of control charts. In this study, it is found that neural network based control chart is suitable to be used as a control chart since the quality of a process can be assured.

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REFERENCES

1. M. V. Koutras, S. Bersimis and P. E. Maravelakis, *Statistical Process Control using Shewhart Control Charts with Supplementary Runs Rules*, Methodology and Computing in Applied Probability, 2007, pp. 207-224.
2. M. B. C. Khoo, *Design of Runs Rules Schemes*, Quality Engineering, 2004, pp. 27-43.
3. R. J. Schalkoff, *Artificial Neural Networks*, New Work: McGraw-Hill Companies, Incorporation, 1997.
4. M. Ibrahim and H. Adnan, *Issues in Development of Artificial Neural Network Based Control Chart Pattern Recognition Schemes,* European Journal of Scientific Research, 2010, pp. 336-355.
5. H. C. Ong, C. H. Leong and S. H. Tai, *A Functional Approximation Comparison between Neural Networks and Polynomial Regression*, WSEAS TRANSACTIONS on MATHEMATICS, 2008, pp. 353-363.
6. C. S. Cheng, *A Multi-layer Neural Network Model for Detecting Changes in the Process Mean*, Computers & Industrial Engineering, 1995, pp. 51-61.
7. C. S. Cheng and S. S. Cheng, *A Neural Network Based Procedure for the Monitoring of Exponential Mean*, Computers & Industrial Engineering, 2001, pp. 309-321.
8. G. A. Barnard, *Control Charts and Stochastic Processes*, Journal of the Royal Statistical Society, 1959, pp. 239-271.
9. A. F. Bissell, *CUSUM Techniques for Quality Control*, Journal of Applied Statistics, 1969, pp. 1-30.
10. R. S. Guh and Y. C. Hsieh, *A Neural Network Based Model for Abnormal Pattern Recognition of Control Charts*, Computers & Industrial Engineering, 1999, pp. 97-108.
11. J. E. Dayoff, *Neural Network Architectures – An Introduction*, New York: Van Nostrand Reinhold, 1990.
12. M. A. Barghash and N. S. Santarisi, *Pattern Recognition of Control Charts using Artificial Neural Networks – Analyzing the Effect of the Training Parameters*, Journal of Intelligent Manufacturing, 2004, pp. 635-644.
13. M. B. C. Khoo and S. H. Quah, *Computing the Percentage Points of the Run-length Distributions of Multivariate CUSUM Control Charts*, Quality Engineering, 2002, pp. 229-310.
14. A. C. Palm, *Tables of Run Length Percentiles for Determining the Sensitivity of Shwehart Control Charts for Averages with Supplementary Runs Rules*, Journal of Quality Technology, 1990, pp. 289-298.